# The Debt Brake Business Cycle and Welfare Consequences of Germany's New Fiscal Policy Rule

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**Abstract** In a New Keynesian DSGE model with non-Ricardian consumers, we show that automatic stabilization according to a countercyclical spending rule following the idea of the debt brake is well suited both to steer the economy and in terms of welfare. In particular, the adjustment account set up to record public deficits and surpluses serves well to keep the level of government debt stable. However, it is essential to design its feedback to government spending correctly, where discretionary lapses should be corrected faster than lapses due to estimation errors. (JEL: E32, G61, E62)

Keywords Fiscal Policy  $\cdot$  Debt Brake  $\cdot$  Welfare  $\cdot$  DSGE

# 1 Introduction

What are the business cycle and welfare consequences if around 20 percent of GDP resulting from public spending are steered according to Germany's new fiscal policy rule? This is the question we address within the paper at hand. The current financial crisis, which was followed by a severe economic downturn, has (again) revealed the importance of sound fiscal policies. In this respect, the necessity for a rule-based framework to guarantee sustainable fiscal policies is acknowledged by many (see, among others, IMF, 2008; Allsopp and Vines, 2005; or Solow, 2005) due to reasons revealed in the political economic

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literature (see, for example, Velasco, 1999, 2000; von Hagen, 1992; Woo, 2005; Stähler, 2009; or Eslava, 2010). Not only in Europe, Germany's new fiscal policy rule, the so called debt brake, is perceived to be a desirable tool for achieving sound fiscal policies and as an amendment to strengthen supranational fiscal policy rules such as the Stability and Growth Pact (SGP), for example.<sup>1</sup>

Similar to the SGP, the debt brake demands structurally balanced budgets and ties cyclically adjusted government spending (including interest on outstanding debt) to cyclically adjusted trend revenues raised by the fiscal authorities. Therefore, government spending acts as an automatic stabilizer since the fiscal authorities are supposed to finance some of their expenditures from deficits in "bad times", while they have to accumulate surpluses in "good times". In addition to this, the debt brake implements a rule-based feedback of deficits / surpluses accumulated by the fiscal authority by booking these on an "adjustment account" and calling for a correction by cutting / raising future government expenditure accordingly. This rule is said to generate a (pre)determined level of debt-to-GDP ratio in the long run (see Danninger, 2002, Müller, 2006, German Council of Economic Experts, 2007, Kastrop and Snelting, 2008, and Kremer and Stegarescu, 2008, for a discussion and a more detailed description of the rule).

The present paper, to the best of our knowledge the first, analyzes business cycle dynamics and welfare effects of the debt brake in a New Keynesian DSGE model and compares them to those of the SGP, a balanced budget rule as well as a debt brake with a more countercyclical stance, i.e. with (automatic) builtin stabilization. It is worth noting at this point that any DSGE model allowing for public debt, in a sense, incorporates some sort of debt brake necessary in this class of models for stability. The present model, however, explicitly addresses a specific spending rule (planned to be) implemented in practice, while the other analyses often assume tax rules to do the job. Hence, the contribution is that we explicitly analyze the effects of the mechanism which has been implemented at the level of the German constitution. Furthermore, given that most of the rules relate spending to trend revenues and the output gap, which is not known in practice but has to be estimated, we explicitly include potential measurement error.

In order to allow for a fair comparison, we assume an adjustment account to exist for all rules under consideration except for the SGP, where we assume a feedback rule derived and estimated by Gali and Perotti (2003). Strict balanced budget rules exist, for example, in some US states (unless they have set up a "rainy day" fund which is, from its basic idea, similar to the debt

<sup>&</sup>lt;sup>1</sup> The German Economic Association (Verein für Socialpolitik) just recently devoted a special issue of its journal "*Perspektiven der Wirtschaftspolitik*" (in German) to the German debt brake. Therein, Feld (2010) and Korioth (2010) discuss the rationale and the effectiveness of the debt brake, Heinemann (2010) assesses its stabilizing effects, Ragnitz (2010) addresses the system of consolidation transfers to the Länder in order to be able to get the debt brake started in 2020, Renzsch (2010) assesses the (general) difficulty of incorporating such systems into federal countries and König et al. (2010) analyze the history of the evolution of the debt brake in the light of political economic arguments.

brake regime; see Rodriguez-Tejedo, 2006), while the debt brake regime with (automatic) built-in stabilization is close to how automatic stabilizers are conventionally modelled in the literature (see Taylor, 2000). Our model is in the manner of Gali et al. (2007) and Leith and Wren-Lewis (2007) with Ricardian and non-Ricardian households, a firm sector with staggered price setting as in Calvo (1983), a monetary authority, for which we assume it follows a simple Taylor rule, and a fiscal authority that implements a debt brake (DB, henceforth), a debt brake with automatic built-in stabilization (AS, henceforth) or a balance budget rule (BB, henceforth), respectively. Our general finding is that a rule which steers fiscal expenditures along the trend path and abstains from activism seems preferable as it smoothes economic developments by preserving fiscal sustainability.

Not surprisingly, all rules fare worse than optimal policy. Nevertheless, as optimal policy does not seem to be implementable in practice, we think analyzing and comparing implementable rules is of importance. The BB potentially destabilizes the economy and gives rise to sunspot equilibria. Due to erratic spending schemes, the balanced budget regime triggers boom-bust cycles in consumption among non-Ricardian households. As monetary authorities do not have leverage on these hand-to-mouth consumers, such a fiscal policy stance may even generate sunspot equilibria if the central bank adopts the Taylor principle (see also Gali, 2004). Accordingly, in our baseline calibration, the overall welfare loss relative to optimal policy, calculated as an average consumer loss function for the aggregated shocks, would increase by 7.23% (2.94%) if fiscal authorities were to switch from a DB a BB excluding (including) measurement error. The SGP demands structurally balanced budgets and, form its basic idea, already ties government spending to trend revenues. Whenever the structural deficit or debt-to-GDP ratios deviate from their reference values, the government is supposed to take action in order to correct for this. By law, there is no explicit rule what this actually means. However, Gali and Perotti (2003) derive such a feedback rule and estimate the corresponding parameters detailing how Germany has implemented this rule in the past. We find that the SGP still generates a strong procyclical stance in contrast to its basic idea of keeping public spending merely close to trend revenues. This can mainly be attributed to the lack of a timely rule-based feedback as it is implemented in the DB regime through the adjustment account. In terms of welfare, the economy's welfare loss would increase by 9.90% (13.29%) if fiscal authorities move from a DB to a SGP regime excluding (including) measurement error.

The DB acts countercyclically by construction in the sense that, as government spending is, in principle, fixed to trend revenues, spending is relatively lower in good times and vice versa. This countercyclical stance is, however, diminished, and we find that government spending is positively – albeit only mildly – correlated with the cyclical fluctuations in GDP. This can be attributed to the interest payments on outstanding debt and the commitment to keeping overall debt constant over time, i.e. to the feedback from the adjustment account. For a shock positively influencing actual government revenue, this implies that these additional funds are gradually spent over time. The AS explicitly necessitates stabilization in output which augments the countercyclical stance compared to the DB. Indeed, we find that government spending in such a regime moves in an opposite direction to the cyclical movements in GDP. But the countercyclical stance of government spending is also only relatively small in the AS regime, which can again be attributed to interest on outstanding debt and the adjustment account as is the case for the DB regime. Hence, the difference between the two regimes lies in the fact that government spending moves with the cycle of GDP in the DB regime and opposite to the cycle of GDP in the AS regime. Both regimes can still be considered countercyclical because of generally keeping government spending to a large extent independent of revenues and, not surprisingly from the construction of the spending rules, differ only in their countercyclical stance.

In terms of welfare, a DB and AS regime are quite comparable as the difference in welfare loss relative to optimal policy is limited to only 2.80% (1.53%) excluding (including) trend estimation errors. Nevertheless, the AS is the DSGE winner because it keeps expenditures a little closer to trend revenues than the DB itself and, therefore, attenuates the adverse effects of government spending on wages as it does not crowd in private consumption as much as the DB. Only if we analyze the welfare effects for each shock separately do we find that, for a cost-push shock, the BB may be the preferable option. The reason is that the cost-push shock boosts inflation while decreasing output and tax revenues, which reduces government spending. The (additional) decrease in aggregate demand and the resulting anti-inflationary stance is found to be welfare enhancing as inflation volatility – the main driver of the welfare loss in this model class – is diminished.

With regard to the adjustment account, we find that the feedback of real government spending should ideally differ with the shock. Discretionary government spending shocks should be corrected as soon as possible, while all other shocks (generating expectation errors) should fade out slowly over time in order to keep those fluctuations actively introduced into the system low. From a qualitative point of view, all our results are not altered when allowing for trend estimation errors which, in practise, are likely to be made. Nevertheless, we note that trend misestimation relatively worsens the performance of those rules tying expenditures to trend. Still, we find that the adjustment account is well suited to prevent debt from dramatically increasing, while equally stabilizing inflation and output whenever the feedback is set optimally in the presence of estimation errors, too. Whenever the feedback is set too low, the economy is subject to more pronounced cycles in GDP and inflation and, thus, welfare losses.

It could be argued that, because we consider public consumption positively influencing households' utility, our welfare calculations are biased towards a rule keeping fluctuations in public spending stable and, therefore, favoring the DB or AS regime. In a way, this is true. Using a spending rule for lump-sum transfers to households instead of public consumption may indeed diminish the beneficial effects of these rules. This is because of (i.) the bias resulting from public consumption in the utility function and because of (ii.) the fact that transfers to households tend to generate lower inflation volatility due to the indirect demand effects via private consumption (through rule-of-thumb households) while public consumption provokes immediate demand-driven inflation volatility. Simulating our model with a full transfer rule (instead of a spending rule), we find that the DB and the AS regime are indeed beaten by the BB regime and the SGP. The reduction in the welfare loss is, however, very small. Furthermore, it has to be borne in mind that the debt brake regimes consider public spending without the autonomously financed social security systems. In Germany, the spending components which can be considered transfers to households in our model (for example, monetary and in kind social benefits and subsidies paid by the federal government) make up only a bit more than 20% of total government spending. Taking this into account and calculating the aggregate welfare loss resulting from a simultaneous change in spending and transfers according to their shares in public spending, we find that the transfer effects do not overcompensate the findings of a rule only considering public consumption. Therefore, we believe that focussing on government consumption entering households' utility in the model presented below is a justifiable simplification.

In literature, recently the discussion about simple stabilizing fiscal rules related to debt, their optimal design and, partly, their strategic interaction with monetary policy has been taken up (see e.g. Kirsanova and Wren-Lewis, 2007, Kirsanova et al. 2005, 2007, and Fragetta and Kirsanova, 2007). Starting with Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2007), the studies discuss optimal fiscal policy (and the interaction with monetary policy) whenever fiscal authorities can commit to a certain policy. In contrast, we explicitly assume that there are no commitment technologies such as commitment under a timeless perspective or optimal Ramsey plans available – also due to the political incentives hinted at earlier. Rather, we assume that fiscal authorities are pledged towards a constant debt-to-GDP ratio in the long run which necessitates a fiscal rule. Thus we exclude by assumption that debt follows a random walk as it is optimal under commitment, and we construct a model that reconciles the reactions of macroeconomic variables to a fiscal policy shock found empirically. Gali et al. (2007) show that this happens in DSGE models with rule-of-thumb consumers as well as sticky prices and deficit financing. Straub and Tchakarov (2007), Leith and Wren-Lewis (2007) and Gali and Monacelli (2008) find that countercyclical fiscal policy – a feature of the DB and automatic stabilizer regime – may be welfare enhancing in such set-ups. The main reason is that such fiscal actions help to at least partly internalize the externalities caused by the implemented rigidities and market imperfection, and to keep fluctuations in inflation and disutility of labor smaller than without stabilization. This paper contributes to the debate by discussing different spending rules in a commonly used macroeconomic model among which some rules are indeed implemented in practice. We further point out which key elements have to be taken into account when designing such rules.

The rest of the paper is organized as follows. Section 2 introduces the model and derives the log-linearized version. In section 3, we analyze the impulse responses of our model, while section 4 contains some welfare considerations. In section 5, we have a look at some important policy issues. Section 6 concludes.

# 2 The model

In this section, we present a New Keynesian model including monetary and fiscal authorities. Firms are categorized into the final goods sector and a continuum of intermediate goods producers. Intermediate good producers face monopolistic competition and prices are set in a staggered way following Calvo (1983). Households obtain utility from consumption, public goods and leisure, and invest in state contingent securities. The household sector is partitioned into Ricardian and non-Ricardian households. The Ricardian households, with share  $(1 - \lambda)$ , own the firms and invest state contingent securities, whereas non-Ricardian households, with share  $\lambda$ , are hand-to-mouth consumers in the sense that they spend their disposable income. Monetary policy follows a Taylor rule. Government expenditures are financed by debt or distortionary taxes levied on wages and consumption. Fiscal policy is implemented by a spending rule incorporating the debt brake, the automatic stabilizer regime or the balanced budget rule. The model is built on the framework of Gali, et al. (2007), Leith and Wren-Lewis (2007), and Mayer and Grimm (2008).

In what follows, any aggregate variable  $X_t$  is defined by a weighted average of the corresponding variables for each consumer type, i.e.,  $X_t = \lambda X_t^r + (1 - \lambda)X_t^o$ , where the superscripts o and r stand for optimizing and ruleof-thumb consumers, respectively. Further, variables with a "bar" (as in  $\bar{X}$ ) define the deterministic steady-state value of X, while variables with a "hat" (as in  $\hat{X}$ ) define percentage deviations from the steady state defined as  $\hat{X}_t = log(X_t/\bar{X}) \approx (X_t - \bar{X})/\bar{X}$ .

## 2.1 Firms and price setting

# 2.1.1 Final goods producers

The final good is bundled by a representative firm that operates under perfect competition according to

$$Y_t = \left[\int_0^1 Q_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right]^{\frac{\epsilon_t}{\epsilon_t - 1}},\tag{1}$$

where  $Y_t$  is the final good,  $Q_t(j)$  are the quantities of intermediate goods, indexed by  $j \in (0, 1)$ , and  $\epsilon_t > 1$  is the time-varying elasticity of substitution in period t. Profit maximization implies the following demand schedule for all  $j \in (0, 1)$ 

$$Q_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} Y_t.$$
(2)

The zero-profit theorem implies  $P_t = \left[\int_0^1 P_t(j)^{(1-\epsilon_t)} dj\right]^{\frac{1}{(1-\epsilon_t)}}$ , where  $P_t(j)$  is the price of the intermediate good  $j \in (0, 1)$ . As in Smets and Wouters (2003), we assume that  $\epsilon_t$  is stochastic. This means that  $\Phi_t = \frac{\epsilon_t}{(\epsilon_t - 1)}$  reflects the timevarying mark-up in the goods market. We get  $\Phi_t = \Phi + \hat{\Phi}_t$ ,  $\hat{\Phi}_t$  is i.i.d. normally distributed. Then,  $\Phi = \frac{\epsilon}{(\epsilon - 1)}$  is the deterministic mark-up in steady state.

### 2.1.2 Intermediate goods producers and prices

Profit by firm j at time t is given by

$$\Pi_t(j) = P_t(j)Q_t(j) - W_t(1 - \tau_n^s)N_t(j),$$
(3)

and  $W_t$  denotes the nominal wage rate and  $N_t$  are labor services rented by firms. The production technology available to firms is linear in labor

$$Q_t(j) = A_t \cdot N_t(j), \tag{4}$$

and  $A_t$  denotes an aggregate productivity shock with A = 1. We assume staggered price setting which implies that only a fraction  $(1 - \theta_P)$  of firms is able to adapt prices, where  $\theta_P$  is the Calvo parameter (see Calvo, 1983). Additionally, firms receive constant employment subsidies  $\tau_n^s$  on gross labor costs  $W_t N_t(j)$ which undoes the distortions associated with monopolistic competition and the tax wedge in the steady state such that we are able to take a second-order approximation around the efficient steady state (see also Gali and Monacellli, 2008, or Leith and Wren-Lewis, 2007, among others). The subsidies are financed by lump-sum taxes levied on optimizing households.

#### 2.2 The household sector

We assume a continuum of households indexed by  $j \in (0, 1)$  of which  $(1 - \lambda)$  households own the assets, and behave Ricardian, whereas the rest  $\lambda$  has a consumption ratio of one, i.e. they are non-Ricardian consumers (in the following also called rule-of-thumb consumers). Assume that any household j has the following lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^i \left( C_t^i(j), L_t^i(j) \right), \tag{5}$$

where i = o, r indicates optimizing and rule-of-thumb households, respectively. The per-period utility function for all households is given by

$$U_t^i(j) = \zeta_t \left[ (1 - \chi) log \left( C_t^i(j) \right) + \chi log \left( G_t \right) + \nu log \left( L_t^i(j) \right) \right], \tag{6}$$

where  $\zeta_t$  is a common preference shock, with  $E{\zeta_t} = \overline{\zeta} = 1$ .  $L_t^i(j)$  is household j's leisure, where  $N_t^i(j) = 1 - L_t^i(j)$  gives the corresponding labor supply of household j. v > 0 measures how leisure is valued compared to consumption  $C_j^i(j)$ .  $\chi \in (0, 1)$  measures the weight given to public goods consumption  $G_t$ . The log-log-log specification of preferences is for the ease the welfare computations later on.

# 2.2.1 Optimizing households

The flow budget constraint of optimizing households in real terms is given by

$$(1+\tau_t^C)C_t^o(j) + \frac{B_{t+1}^o(j)}{P_t} - T_t^{s,n} \le (1-\tau_t^d)\frac{W_t}{P_t}N_t^o(j) + \frac{\Pi_t^o(j)}{P_t} + \frac{B_t^o(j)}{P_t}R_{t-1},$$
(7)

where  $B_{t+1}$  denotes a bond issued by the government. The bond pays a gross interest equal to the risk-free nominal rate  $R_t$ , which is monetary authority's policy instrument.  $W_t$  is the nominal wage rate. As productivity of Ricardian and non-Ricardian consumers are identical and labor services offered to firms are perfect substitutes, we can drop the superscript o and r in the following regarding wages.  $\Pi_t^o(j)$  are nominal profits from the intermediate goods sector.  $\tau_t^d$  is a distortionary tax rate levied on nominal labor income, while  $\tau_t^C$  is a consumption (quasi-value added) tax.  $T_t^{s,n}$  denotes the lump-sum tax levied on optimizing households to finance the constant employment subsidy  $\tau_n^s$ .

Optimizing households maximize utility, equation (5) with respect to consumption, leisure and bond holdings subject to the intertemporal version of the budget constraint, equation (7). We find that:

$$\frac{\zeta_t}{C_t^o(j)} = \beta R_t E_t \left\{ \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \cdot \frac{\zeta_{t+1}}{C_{t+1}^o(j)} \cdot \frac{P_t}{P_{t+1}} \right\}$$
(8)

is the consumption Euler equation for optimizing households and derive

$$\frac{C_t^o(j)}{L_t^o(j)} = \frac{(1-\chi)}{\upsilon} \cdot \frac{(1-\tau_t^d)}{(1+\tau_t^C)} \cdot w_t$$
(9)

as the labor supply schedule, where  $w_t = \frac{W_t}{P_t}$  and  $L_t^o(j) = [1 - N_t^o(j)]$ .

## 2.2.2 Rule-of-thumb consumers

The lifetime utility of rule-of-thumb consumers is also given by equations (5) and (6). However, as they do not have access to capital markets, their budget constraint is static:

$$(1 + \tau_t^C)C_t^r(j) = (1 - \tau_t^d)\frac{W_t}{P_t}N_t^r(j).$$
(10)

Hence, rule-of-thumb consumers maximize equation (5) with respect to  $C_t^r(j)$  and  $L_t^r(j)$  subject to equation (10).

$$\frac{C_t^r(j)}{L_t^r(j)} = \frac{(1 - \tau_t^d)}{(1 + \tau_t^C)} \frac{(1 - \chi)}{\upsilon} w_t,$$
(11)

which, substituted in equation (10) and remembering that  $N_t^r = 1 - L_t^r$ , yields

$$N_t^r(j) = \frac{(1-\chi)}{(1-\chi)+\nu} \iff L_t^r(j) = \frac{\nu}{(1-\chi)+\nu}.$$
 (12)

Hence, labor supply by rule-of-thumb consumers is constant. Using equation (12) and equation (11), we find that consumption is a function of disposable income.

$$C_t^r(j) = \frac{(1-\chi)}{(1-\chi)+\upsilon} \cdot w_t \cdot \frac{(1-\tau_t^d)}{(1+\tau_t^C)}.$$
(13)

## 2.3 Fiscal authorities

The government issues bonds  $B_{t+1}$ , and collects consumption taxes  $\tau_t^C P_t C_t$ as well as labor taxes  $\tau_t^d W_t N_t$ . The receipts are used to finance government expenditure  $P_t G_t$  and interest on outstanding debt  $R_{t-1}B_t$ . Furthermore, the government has to pay subsidies on labor costs for which it collects lump-sum taxes. The government's flow budget constraint reads:

$$B_{t+1} + \tau_t^d W_t N_t + \tau_t^C P_t C_t + T_t^{s,n} = R_{t-1} B_t + P_t G_t + \tau_n^s W_t N_t.$$
(14)

At each point in time, it holds that  $\tau_n^s W_t N_t = T_t^{s,n}$  such that it cancels out in equation (14) (see Leith and Wren-Lewis, 2007). In the following we rewrite the budget constraint in terms of the cyclically adjusted balance. In a first step we express equation (14) in real terms and normalize by  $\bar{Y}$ :

$$\frac{B_{t+1}}{P_t\bar{Y}} + \frac{\tau_t^d w_t N_t}{\bar{Y}} + \frac{\tau_t^C C_t}{\bar{Y}} = \frac{R_{t-1}B_t}{P_{t-1}\bar{Y}} \frac{P_{t-1}}{P_t} + \frac{G_t}{\bar{Y}}.$$
(15)

Defining  $\tilde{b}_t = \frac{B_t}{P_{t-1}Y}$  as the cyclically adjusted debt, and government tax revenues as  $\Psi_t = \tau_t^d W_t N_t + \tau_t^C P_t C_t$ , where

$$\frac{\Psi_t}{P_t\bar{Y}} = \frac{\tau_t^d w_t N_t}{\bar{Y}} + \frac{\tau_t^C C_t}{\bar{Y}},\tag{16}$$

equation (15) rewrites to

$$\tilde{b}_{t+1} + \frac{\Psi_t}{P_t \bar{Y}} = R_{t-1} \tilde{b}_t \frac{P_{t-1}}{P_t} + \frac{G_t}{\bar{Y}}.$$
(17)

which leads us to the cyclically adjusted debt from its steady state:

$$b_t = \tilde{b}_t - \bar{b} = \frac{B_t}{P_{t-1}\bar{Y}} - \frac{\bar{B}}{\bar{P}\bar{Y}}$$
(18)

In what follows, we will describe the different fiscal spending rules in more detail. First, describe an extreme case of a sustainable fiscal rule, namely the balanced budget rule in which the government is not allowed to spend more than the projected funds raised. Second, we turn to a spending rule currently in place, the SGP. Last, we turn to the rules this paper mainly focusses on, the debt brake and the rule with additional rule-based stabilization.

# 2.3.1 The balanced budget rule

As a benchmark for a sustainable spending rule, we introduce a balanced budget, which implies that the government is not allowed to spend more than the projected funds raised. To allow for a fair comparison with the debt brake rules, we assume that any expectation errors, i.e. differences between projected and actual funds raised and discretionary spending shocks  $g_t$  are booked on the adjustment account  $AC_t$  to record lapses in spending behavior. Thus spending according to the balanced budget rule is determined by projected revenues minus previous balances booked on the adjustment account, i.e.  $E_{t-1}\{\Psi_t\} - \rho \cdot AC_{t-1}$ , where  $\rho \in [0, 1]$  is a partial adjustment parameter indicating how much of an effect earlier lapses in the spending behavior have on current spending. This implies that actual (ex-post) spending is given by  $(R_{t-1}-1)B_t + P_tG_t = E_{t-1}\{\Psi_t\} - \rho AC_{t-1} + g_t$ . The adjustment account reads  $AC_t = (1-\rho)AC_{t-1} + \nu_t + E_{t-1}\{\Psi_t\} - \Psi_t$ . Government spending can be normalized in real terms as follows:

$$(R_{t-1}-1)\tilde{b}_t + \frac{G_t}{\bar{Y}} = \underbrace{E_{t-1}\left\{\frac{\Psi_t}{P_t\bar{Y}}\right\} - \rho \cdot \frac{P_{t-1}}{P_t} \cdot ac_{t-1}}_{=Rule-based spending} + \frac{g_t}{P_t\bar{Y}}$$
(19)

and, for the adjustment account,

$$ac_{t} = (1-\rho)\frac{P_{t-1}}{P_{t}} \cdot ac_{t-1} + \frac{g_{t}}{P_{t}\bar{Y}} + \underbrace{E_{t-1}\left\{\frac{\Psi_{t}}{P_{t}\bar{Y}}\right\} - \frac{\Psi_{t}}{P_{t}\bar{Y}}}_{Expectation\ error},$$
(20)

where  $ac_t = \frac{AC_t}{P_t Y}$ .

# 2.3.2 The Stability and Growth Pact

Nominal deficit in one period is given by  $D_{t+1} \equiv B_{t+1} - B_t = P_t G_t - \Psi_t + (R_{t-1} - 1)B_t$ . Normalizing by  $\bar{Y}$  and defining  $\tilde{d}_t = D_t/(P_{t-1}\bar{Y})$  as we did with nominal debt, this yields  $\tilde{d}_{t+1} = \frac{G_t}{Y} - E_{t-1}\left\{\frac{\Psi_t}{P_tY}\right\} + (R_{t-1} - 1)\frac{P_{t-1}}{P_t}\tilde{b}_t$  in (normalized) real terms. The structural deficit  $\tilde{d}_t^*$  is given by the deficit adjusted for automatic responses of revenues resulting from deviations of the bases of revenue components from their steady-state, i.e. trend values (see, for example, Bilbiie et al., 2008). We know that trend values are not known to the government and must be estimated. Therefore, the trend expenditure component is likely to be subject to significant errors in real time.<sup>2</sup> In order to account for this, we introduce potential measurement error in the spending rule, where  $Z_{\Psi,t}$  with  $E_t\{Z_{\Psi,t}\} = 1$  represents the estimation error for trend

 $<sup>^2\,</sup>$  Brunez (2003), Döpke (2006) and Kremer and Stegarescu (2008) provide evidence for estimation errors being an issue. Heinemann (2006) even suggests that politicians may have an incentive to misestimate.

revenues to be described in more detail in section 3.2.2. Hence, structural deficits evolve according to

$$\tilde{d}_{t+1}^* = \tilde{d}_{t+1} - \left[ E_{t-1} \left\{ \frac{\Psi_t}{P_t \bar{Y}} \right\} - \frac{\bar{\Psi}}{\bar{P} \bar{Y}} \cdot Z_{\bar{\Psi}, t} \right] \\
= \frac{G_t}{\bar{Y}} - \frac{\bar{\Psi}}{\bar{P} \bar{Y}} \cdot Z_{\bar{\Psi}, t} + (R_{t-1} - 1) \frac{P_{t-1}}{P_t} \tilde{b}_t.$$
(21)

The SGP demands structurally balanced budgets and implies that, whenever the structural deficit-to-GDP ratio and/or the debt-to-GDP ratio deviate from their reference values (i.e. the 3% and 60% criteria, respectively), the government is supposed to take action in order for these values to move back to target. Letting  $d_t = \tilde{d}_t^* - \bar{d}^*$  denote a first-order Taylor approximation of  $\tilde{d}_t^*$ around its steady state, we follow Gali and Perotti (2003), who derived and estimated a fiscal policy rule consistent with the SGP, and assume that the structural deficit is adjusted according to the following rule:

$$d_{t+1} = \tilde{d}_{t+1}^* - \bar{d}^* = \phi_d \cdot d_t + \phi_b \cdot b_t + \phi_y E_{t-1} \left\{ \hat{Y}_t \right\},$$
(22)

where  $\hat{Y}_t$  is a first-order Taylor approximation of  $Y_t$ . Rules of this type have been studied extensively, including by Bohn (1998) and Bilbiie et al. (2008). The parameter  $\phi_d$  captures the possibility that budget decisions are autocorrelated, while the parameter  $\phi_b$  determines the response of the deficit to the beginning-of-period ratio of debt to GDP, hence capturing a "debt stabilization" motive: a negative value of  $\phi_b$  indicates that deficits are adjusted in order to stabilize outstanding debt.  $\phi_y$  measures the possibility for a countercyclical fiscal stance for  $\phi_y < 0$  (a procyclical stance for  $\phi_y > 0$ ). Estimates for these parameters for the Euro area and Germany are calculated in Gali and Perotti (2003). In order for the fiscal rule of equation (22) to be met in our model setup, we assume that the government adopts a spending rule such that  $\tilde{d}^*_{t+1}$ of equations (21) and (22) follow the same path. Corsetti et al. (2009, 2010) discuss the implications of a similar spending rule that results from such a proceeding.

#### 2.3.3 The debt brake and additional rule-based stabilization

The main idea of the debt brake rule is that real spending including interest on outstanding real debt, i.e.  $(R_{t-1}-1)\frac{B_{t+1}}{P_t} + G_t$ , must be equal to real trend revenues, i.e.  $\frac{\bar{\Psi}}{P}$ , which builds in a countercyclical fiscal stance as surpluses arise in "good times" and deficits in "bad times". Within the built-in automatic stabilizer regime, government spending has, on top, an additional countercyclical spending component (see e.g. Taylor, 2000; Artis and Buti, 2000; or Buti et al., 2001). In order to make this rule comparable to the debt brake, we assume that, in the steady-state, both rules are tied to steady-state revenues. However, the higher countercyclical stance of the automatic built-in stabilizer augments the rule-based spending by  $E_{t-1} \{(\bar{Y}/Y_t)^{\alpha}\}$ , where  $\alpha > 0$  captures the magnitude of the stance taken by the rule. This implies relatively more spending in expected "bad times",  $Y_t < \bar{Y}$ , and vice versa. Thus:

$$(R_{t-1}-1)\tilde{b}_t + \frac{G_t}{\bar{Y}} = \underbrace{\frac{\bar{\Psi}}{\bar{P}\bar{Y}} \cdot Z_{\bar{\Psi},t} \cdot E_{t-1} \left\{ \left(\frac{\bar{Y}}{Y_t} \cdot \frac{1}{Z_{Y,t}}\right)^{\alpha} \right\} - \rho \cdot \frac{P_{t-1}}{P_t} ac_{t-1}}_{=Rule-based \ spending} + \underbrace{\frac{g_t}{P_t\bar{Y}}}_{(23)}$$

It holds that  $\alpha = 0$  for the debt brake and  $\alpha > 0$  for the automatic stabilizer. Again,  $Z_{\Psi,t}$  is the estimation error in trend revenues, while  $Z_{Y,t}$  indicates that the government, when planning to adopt a countercyclical stance, may under or overestimate the cyclical component of GDP, too.

Regarding the adjustment account, we know that a discretionary spending shock  $g_t$  must reduce future spending as in the case for the balanced budget. As the debt brake ties spending to trend revenues, any deficit resulting from deviations of true revenues from trend revenues have to be repatriated in future periods and the adjustment account books  $\frac{\bar{\Psi}}{PY} - \frac{\Psi_t}{P_tY_t}$ . For the automatic built-in stabilizer regime additional spending  $E_{t-1}\left\{\left(\bar{Y}/Y_t\right)^{\alpha}\right\}$  have to be booked on the adjustment account in order to generate a constant level of debt. Furthermore, we assume that the government needs only one period to detect the real time measurement error. Whenever the government obtains tax revenues in one period, it can, thus, deduct the committed estimation error made in the previous period. It thus formally holds that

$$ac_{t} = (1 - \rho) \cdot \frac{P_{t-1}}{P_{t}} \cdot ac_{t-1} + \frac{g_{t}}{P_{t}\bar{Y}} + \underbrace{\varrho\left[\frac{\bar{\Psi}}{\bar{P}\bar{Y}} - \frac{\Psi_{t}}{P_{t}\bar{Y}}\right]}_{Booking \ DB; \ \alpha = 0, \ \varrho = 1} + \frac{\bar{\Psi}}{\bar{P}\bar{Y}} Z_{\Psi,t-1} + \frac{\bar{\Psi}}{\bar{P}\bar{Y}} \cdot \underbrace{\left[E_{t-1}\left\{\left(\frac{\bar{Y}}{Y_{t}}\right)^{\alpha} \cdot \left(\frac{1}{Z_{Y,t-1}}\right)^{\alpha}\right\} - 1\right]}_{Booking \ AS; \ \alpha > 0, \ \varrho = 0}, \qquad (24)$$

where  $\rho = 1$  for the debt brake and  $\rho = 0$  for the automatic stabilizer. Remember that negative entries in the adjustment account  $ac_t$  stand for assets/surpluses. Note that, as the government is committed towards keeping real debt constant in the long run, debt services and the adjustment account can almost cancel out the automatic stabilizer component such that the fiscal stance might only move moderately countercyclical to GDP.

#### 2.4 Market clearing

In clearing of factor and goods markets, the following conditions are satisfied

$$Y_t = C_t + G_t, \tag{25}$$

where  $C_t = \lambda C_t^r + (1 - \lambda) C_t^o$  is aggregate consumption. Furthermore,

$$Y_t(j) = Q_t(j) \tag{26}$$

and (in per capita units)

$$N_t = \frac{1}{\lambda} \int_0^\lambda N_t^r(j) dj + \frac{1}{(1-\lambda)} \int_\lambda^1 N_t^o(j) dj.$$
(27)

**Proposition 1** Define a linear combination of variables  $ac_t = (1-\rho)ac_{t-1}+\eta_t$ representing the linearized version of the adjustment account, where  $\eta_t$  is a set of orthogonal white noise error terms driving the economy (i.e.  $\eta_t$  represents presents a linear combination of the shocks in our model). Then, the adjustment account will be non-stationary if  $\rho = 0$  across all regimes incorporating the adjustment account.

*Proof* By backward induction, it holds that

$$ac_t = (1-\rho)^{\infty} ac_{t-\infty}^{\infty} + \sum_{k=0}^{\infty} (1-\rho)^k \eta_{t-k},$$

where  $\eta_t$  is a white noise process. It then holds that  $ac_t$  will be stationary only if  $0 < |\rho| < 1$ , as all sums are bounded.

Proposition 1 states that even if shocks are symmetrically distributed, they will not cancel each other out over the business cycle such that  $ac_t$  will be a non-stationary variable for  $\rho = 0$ . Thus, the pure existence of exceptional errors is sufficient to justify a partial feedback from the adjustment account to government spending as business cycle dynamics will not render  $ac_t$  stationary by itself. This result is important because, in the political debate, there seems to be the conjecture that a sustainable fiscal policy is a necessary and sufficient condition for stationarity – which is not the case. The discussion relates to Schmitt-Grohé and Uribe (2003), who show that a similar stationarity-problem arises in small open economy models with incomplete asset markets likewise necessitating some feedback to assets held by the small open economy in order to close the model.

**Proposition 2** All variables in steady state and, thus, welfare can be expressed by deep parameters and fixed levels of tax rates  $\bar{\tau}^d$  and  $\bar{\tau}^C$ . The steady states are identical across all fiscal regimes considered.

Proof See Appendix F.

Proposition 2 states that the steady-state levels of all variables are identical across fiscal regimes. This is of utmost importance for our welfare exercise as it allows us to focus on the business cycle implications of fiscal policy, whereas we do not need to adjust our conclusions for differences in the steady states.

# 2.5 Shocks

The shocks are assumed to follow a univariate autoregressive process of the form  $\zeta_t = \rho_{\zeta} \cdot \zeta_{t-1} + \tilde{\epsilon}_{\zeta,t}$ ,  $A_t = \rho_A \cdot A_{t-1} + \tilde{\epsilon}_{A,t}$ ,  $\epsilon_t = \rho_\epsilon \cdot \epsilon_{t-1} + \tilde{\epsilon}_{\epsilon,t}$ ,  $v_t = \rho_v \cdot v_{t-1} + \tilde{\epsilon}_{v,t}$ ,  $z_t = \rho_z \cdot z_{t-1} + \tilde{\epsilon}_{z,t}$ ,  $g_t = \rho_g \cdot g_{t-1} + \tilde{\epsilon}_{g,t}$ ,  $\xi_t = \rho_{\xi} \cdot \xi_{t-1} + \tilde{\epsilon}_{\xi,t}$ ,  $Z_{\Psi,t} = \rho_{\Psi} \cdot Z_{\Psi,t-1} + \tilde{\epsilon}_{\Psi,t}$  and  $Z_{Y,t} = \rho_Y \cdot Z_{Y,t-1} + \tilde{\epsilon}_{Y,t}$  where  $\tilde{\epsilon}_{j,t}$  with  $j \in \{\zeta, A, \epsilon, v, z, g, \xi, \Psi, Y\}$  represents random i.i.d. shocks. Hence, the log-linearized version of the equilibrium equations just described as well as the shock rules are able to describe the cyclical behavior of the economy.

#### 3 Calibration and impulse response analysis

In this section, we provide details on the business cycle dynamics of the different fiscal rules. Because the effects coming from the fiscal side of the model are invariant with respect to the specific shock, we concentrate on a consumer preference shock as well as trend misestimation. For all other shocks, the reader is referred to an extended working paper version (see Mayer and Stähler, 2009).

#### 3.1 Calibration strategy

We choose parameter values typically recommended to describe the euro area. We set tax rates, such that the level of public to private consumption is close to three. The labor tax rate is set to  $\bar{\tau}^d = 0.10$ . The consumption tax rate is calibrated to  $\bar{\tau}^C = 0.18$  (see Coenen, Mohr and Straub, 2008). This determines the private consumption to output ratio and the government consumption to output ratio which are equal to  $\gamma_C = 0.74$  and  $\gamma_G = 0.26$ . Recall, the debt brake does not encompass the social security system, which would call for higher values of  $\gamma_G$ .

For the fraction of liquidity constraint consumers, we choose  $\lambda = 0.33$ , which engineers a moderate crowding out of private consumption to a highly correlated expenditure shock on impact. For moderately autocorrelated spending shocks, this can replicate a crowding in of private consumption, which is in line with evidence reported from a VAR by Gali et al. (2007). For lower values of  $\lambda$  as, for instance, proposed by Coenen, McAdam and Straub (2008), our model would still predict a substantial crowding out in private consumption which might be considered counterfactual. Additionally there is overwhelming theoretical and empirical evidence to suggest that fiscal multipliers are significantly different from zero and in the neighborhood of one (see e.g. Baxter and King, 1993, Fatas and Mihov, 2001, Blanchard and Perotti, 2002, Perotti, 2005, Heppke-Falk et al., 2010). A sizable fraction of rule of thumbers also helps along this dimension to reconcile the model with empirical evidence.

Since we do not have a distinctive imagination for appropriate numerical values for  $\rho$ , which governs the partial feedback from the adjustment account to expenditures, we choose the parameter such that our welfare, which is discussed in section 4, is maximized. We find in particular that, for all shocks

except for a government expenditure shocks, the algorithm prefers small parameters for  $\rho$ . Accordingly, we set  $\rho = 0.05$ , which generates a unique and determined rational expectations equilibrium (see Appendix A for details). The countercyclical stance in the AS regime is set to  $\alpha = 0.33$  following Taylor (2000). For the SGP, we set  $\phi_y = 0.32$ ,  $\phi_b = -0.02$  and  $\phi_d = 0.46$  according to Gali and Perotti (2003). Also, Corsetti et al. (2009, 2010), who basically use a similar spending rule as we do, have a similar parameterizations.

For the supply side of the model to imply a substantial degree of nominal rigidities, we set  $\theta_p = 0.75$ , which implies that prices are fixed on average for four quarters. This is calibrated somewhere in the middle of the range typically reported in the literature. Coenen, McAdam and Straub (2008) estimate an average price duration of ten quarters using full information Baysian estimation techniques, while Del Negro et al. (2005) only report an average price duration of three quarters. Micro-data for the euro area on price setting report low price durations with a median of around 3.5 quarters (see Alvaraez et al., 2006 for a summary of recent micro-evidence). The steady-state mark-up of intermediate goods producers over marginal cost is set at 10 per cent, implying that  $\epsilon = 11$ .

Following Gali et al. (2007), we specify the household sector similar (i.e. a log-utility function). We set the inverse of the Frisch elasticity of labor supply equal to  $\varphi = 1$ . The discount factor is fixed to  $\beta = 0.99$ , implying a 4% steady-state real interest rate.

The Taylor rule coefficients display values in line with Schmitt-Grohé and Uribe (2007). The inflation coefficient is set to  $\phi_{\pi} = 3.0$ , while for the output gap coefficient, we opt for  $\phi_Y = 0.25$  (see Del Negro et al., 2005; Coenen, Mc Adam and Straub, 2008; and Smets and Wouters, 2003). We set the inflation coefficient to a somewhat higher value than originally proposed by Taylor (1993) as, in the light of rule-of-thumb consumers, the central bank is forced to follow a more anti-inflationary policy. Additionally, Schmitt-Grohé and Uribe (2007) report evidence that values well above 1.5 are welfare enhancing in economies with nominal frictions and set  $\phi_{\pi} = 3.0$ . The interest rate smoothing coefficient is set to  $\mu = 0.85$ .

The autoregression coefficients of the shock processes are set as follows:  $\rho_{\zeta} = 0.882$ ,  $\rho_{\epsilon} = 0.89$ ,  $\rho_z = 0.15$  and  $\rho_A = 0.822$ . These values reflect coefficients found in Coenen, McAdam and Straub (2008) and Smets and Wouters (2003, 2007). For the case of the fiscal spending shock, the recent literature has not yet found a clear-cut consensus. While some authors report evidence for highly autocorrelated fiscal expenditure shocks such as Smets and Wouters (2004) with  $\rho_g = 0.956$ , Chari et al. (2007) attribute only a small role to fiscal expenditure shocks. Others estimate DSGE models and remain tacit on the role of fiscal expenditure shocks by not specifying them (Coenen, McAdam and Straub, 2008). For the trend estimation error shocks, we set  $\rho_{\Psi} = 0.56$ and  $\rho_Y = 0.95$ . A detailed description how we derived these values is relegated to section 3.2.2 because the importance of trend misestimation is addressed there in more detail. An overview of the parameters is found in Table 1, while Table 2 provides an overview of the standard deviation of shocks.

Parameter	Symbol	Value
Discount factor	$\beta$	0.990
Elasticity of demand in intermediate goods sector	$\epsilon$	11.000
Taylor rule coefficient: inflation	$\phi_{\pi}$	3.000
Taylor rule coefficient: output	$\phi_Y$	0.250
Taylor rule coefficient: interest rate smoothing	$\mu$	0.850
Feedback of adjustment account to spending	$\rho$	0.050
Countercyclical stance in the AS regime	$\alpha$	0.330
Autocorrelation in deviations of deficit-to-GDP ratio in SGP	$\phi_d$	0.460
Feedback to deviations in debt-to-GDP ratio in SGP	$\phi_b$	-0.020
Feedback to deviations in output gap in SGP	$\phi_y$	0.320
Fraction of firms that leave their price unchanged	$\theta_p$	0.750
Share of liquidity constraint consumers	$\hat{\lambda}$	0.330
Steady-state rate of employee wage taxes	$ar{ au}^d$	0.100
Steady-state rate of consumption taxes	$\bar{\tau}^C$	0.180
Autoregressive parameter for consumer preference shock	$\rho_{\zeta}$	0.822
Autoregressive parameter for technology shock	$\rho_A$	0.828
Autoregressive parameter for supply shock	$ ho_\epsilon$	0.890
Autoregressive parameter for monetary policy shock	$\rho_z$	0.150
Autoregressive parameter for government spending shock	$ ho_g$	0.956
Autoregressive parameter for trend output estimation shock	$\rho_Y$	0.917
Autoregressive parameter for trend revenue estimation shock	$\rho_{\Psi}$	0.976
Relative weight of leisure to consumption	v	1.000

 Table 1
 Baseline calibration

Table 2 Standard deviations of shocks

Shock type	Standard deviations
Consumer preferences	0.324
Technology	0.628
Price mark-up	0.140
Monetary policy	0.240
Government expenditure	0.331
Government revenue	0.329
Trend revenue estimation error	0.107
Estimation error in output gap	0.597

## 3.2 Impulse response analysis

Given the above calibration, we start off by analyzing the different sets of fiscal policy rules. We will first analyze the different rules neglecting trend estimation errors in order to get a better understanding of the resulting differences in section 3.2.1. Then, in section 3.2.2, we show how trend estimation errors affect the cyclical behavior of the economy and describe what happens in the new DB and AS regimes when the government is subject to such errors. Therein, we also detail the how estimation errors are calibrated. A welfare analysis, including trend estimation errors, can be found in section 4.

## 3.2.1 Analyzing a consumer preference shock

Figure 1 portrays the dynamic response of selected variables to a shock to consumer preferences when the government follows the DB regime. For the sake of space the reader is referred to an extended working paper version for a discussion of the other shocks (see Mayer and Stähler, 2009). Remember that, regarding the effects of the different policy regimes coming from the fiscal side, the discussion is analogous.



Fig. 1 Debt brake and CPS

After a consumer preference shock, firms that are allowed to reset prices increase these to cushion the increasing marginal cost pressure stemming from higher wages to incite households to work more in order to satisfy the additional demand. The increase in real wages, in turn, encourages non-Ricardian consumers to increase their consumption expenditures. Although they only account for one third of the household sector, they drive, on impact, almost 50%in the consumption dynamics and start to dominate the picture. As monetary authorities are determined to dampen inflation variability, they increase real interest rates and slow down consumption expenditures such that inflation falls quickly. The somewhat tough stance on inflation and the implied high interest rate along the adjustment path almost completely wipe out the positive impact of the consumer preference shock for Ricardian households from quarter three onward. The impulse responses portray that fiscal authorities keep expenditures largely stable over the cycle. In particular, the additional funds raised due to an increase in labor and consumption taxes are not spent but passed through to debt. Thus the DB embodies automatic stabilization on the revenue side as government expenditures are decoupled from cyclical movements in revenues and kept at trend. The mildly procyclical movement in government expenditures can be attributed to interest rate payments on outstanding debt and the commitment of fiscal authorities to keep overall debt constant in the long run, which means that the additional funds are spent gradually over time. This is engineered by a low feedback from the adjustment account to government expenditures.



Fig. 2 Automatic stabilizer and CPS

Figure 2 illustrates the response to a consumer preference shock if the government tries to implement the automatic stabilizer rule. As for the case of a debt brake, the additional revenues are not spent but passed through to debt. Additionally, the higher countercyclical stance starts to take effect and diminishes government spending relative to the debt brake regime, which can be seen in the upper right panel. This implies that, in contrast to the pure debt brake regime, where the feedback from the adjustment account and debt services crowd out the countercyclical stance, a slightly countercyclical stance is present in the first three periods of the automatic stabilizer regime. As the government implements spending cuts in periods two and three, although revenue increases are high, surpluses accumulate faster than under the debt brake regime. Only afterwards, debt services and the adjustment account are strong enough to overcompensate the higher countercyclical stance. Besides this, the business cycle dynamics of the debt brake and the automatic stabilizer are very similar.

Figure 3 depicts the business cycle dynamics if fiscal authorities are determined to balance the budget in each period. Due to the planning horizon of one period, the budget will not be balanced in the first period as the unexpected tax revenues are not accounted for in the predetermined government expenditure plans. The regime shift leads to a number of remarkable changes in the business cycle. First, government expenditures become the driving compo-



Fig. 3 Balanced budget and CPS

nent of GDP quantitatively, whereas for the debt brake, private consumption expenditures dominated the picture over the first five quarters. From period two onward, the government spends the additional tax revenues, which has two effects on the economy. On the one hand, firms have to pay significantly higher wages to optimizing households to extent their hours worked while, on the other hand, the significantly higher wages lead to a boom in consumption among liquidity constraint consumers. Accordingly, compared to a debt brake, we observe a somewhat higher inflation rate and higher interest rates The latter almost completely crowd out the consumption expenditures of Ricardian households. The low feedback from the partial adjustment account to expenditures gradually reduces the surpluses accumulated in the first period due to the expectations error.

In Figure 4, we see the business cycle dynamics when governments follow the SGP. In contrast to the balanced budget regime, government spending is not the driving component of GDP from the beginning but, gradually becoming more important. The structural deficit increases. Both can mainly be attributed to the procyclical stance (i.e. a positive  $\phi_y$ ) found by Gali and Perotti (2003) implying that the positive impact of a consumer preference shock on GDP is additionally fueled by an increase in government demand. Given the low but negative feedback from debt to spending (i.e.  $\phi_b$ ), this is not compensated for by the fiscal feedback rule of the SGP. Government spending only gradually falls due to the higher level of deficits. This implies that in the SGP – at least as it has been conducted – the procyclical stance is not as strong on impact as it is in the balanced budget regime, but it lasts longer. Even though the basic idea of the SGP is to tie spending to structural revenues, this does not seem to be sufficient for policy makers to follow such a path according to the estimates of Gali and Perotti (2003). We conclude that, therefore, the



Fig. 4 Stability and Growth Pact and CPS

adjustment account installed in the DB and AS regimes helps to account for earlier spending deviations. This indeed drives spending behavior to the desired path because it implements a rule-based feedback absent in the current SGP regime. Hence, this adjustment account can, from the business cycle perspective, be interpreted as the main improvement to keep structural deficits stable.

## 3.2.2 Cyclical effects of trend estimation errors

The fundamental idea of Germany's new fiscal policy rule is to tie expenditures towards trend revenues. Unfortunately, trend revenues and the state of the business cycle are not perfectly known to the government and must be estimated in real time. Therefore, the trend expenditure component and the discretionary stance to the state of the business cycle are likely to be subject to significant errors in real time.

Based on the real-time dataset of the Deutsche Bundesbank including realtime data for GDP and the Bundesbank's own real time estimates of potential output, the latter directly affecting trend revenues more or less one-to-one, Gerberding et al. (2005) construct time series to quantify the measurement error in the level of the output gap and output potential. As a cross-cheque, we also construct proxies for the level of the output gap and potential output noise based on the real-time data provided by the Federal Reserve Bank of Philadelphia. The data consists of different vintages of GDP series. We get an estimate of the output gap and potential output by using an Hodrick-Prescott (HP) filter with a smoothing parameter of 1600. Repeating this procedure for each vintage, we obtain a series of estimated real-time output gaps and output potentials. Finally, to obtain an estimate of the real-time error, we calculate the difference between final revised measures and the real-time data sets. This gives us an indication on the size of the measurement error. Although this procedure is rather mechanical and does not encompass any judgement of policymakers at the time, the so constructed series are remarkably close to those provided by Orphanides (2004) who uses data on the macroeconomic outlook prepared for meetings of the Federal Open Market Committee.

With these data series at hand we can explore to what extend the government can become a source of destabilization if it responds to the estimated output gap and trend revenues in the presence of measurement error. That is, by responding to noisy real-time estimates of the output gap and potential output, fiscal authorities amplify the noise in the data and thereby generate output fluctuations. Given the shock process for estimation errors detailed in section 2.5, this proceeding yields the AR(1) coefficients reported in Table 1 and the standard deviations reported in Table 2. In line with the Bundesbank's own real time estimates of output potential as reconstructed by Gerbeding et al. (2005), we can also present evidence that the two data series are not orthogonal. By definition, the concept of the output gap is linked to output potential. In the sample at hand, the two series are negatively correlated with  $\rho_{\hat{Z}_Y,\hat{Z}_{\Psi}} = -0.34$  for US-data. A negative correlation could be expected as an overestimation of potential, thus a negative measurement error  $(y_{Pot}^{final} - y_{Pot}^{revised})$  goes by definition hand in hand with an overestimation of the output gap, thus a positive measurement error  $(\hat{y}^{final} - \hat{y}^{revised})$ . Therefore it is natural to expect the two noise process to be negatively correlated. When conducting simulations for the AS-regime we take care of this correlation by drawing from the relevant linear combination of the two series which is defined as:  $\hat{Z}_{\Psi,t} - \alpha \hat{Z}_{Y,t}$ .

Fig 5 portrays what the economy looks like under the DB regime if fiscal authorities overestimate trend revenues in real time by a standard deviation of the measurement shock in potential output. On impact, GDP and consumption start to increase which fuels inflation and leads to higher real interest rates along the adjustment path. As government spending is larger than the (true) additional tax revenues, debt starts to increase and, thus, future spending needs to be cut accordingly.

A comparison to Fig 6 directly reveals that augmenting the fiscal policy rule by a discretionary countercyclical component seriously deteriorates fiscal performance. Fiscal authorities do not only spend more because they belief on error that output potential has increased, but they simultaneously belief that the output gap is negative which triggers additional spending. Therefore the impact of noisy estimates of the gap and potential in conjunction increase debt by twice as much as under a pure debt brake regime.

Thus, the presence of problems in estimation real-time output and potential may weaken the beneficial effects of a countercyclical stance in public finances. In the next section, we will come back to this point from a welfare perspective and analyze whether the existence of real-time errors justify to discard a (strong) countercyclical factor. Put differently, we ask the question



Fig. 5 Debt brake and measurement error



Fig. 6 Automatic stabilizer and measurement error

whether the (generally found) advantages of a countercyclical component can outweigh its disadvantages in the presence of noisy real-time estimates.

# 4 Welfare

In this section we aim to establish a welfare ranking between the different fiscal policy regimes. As shown in Appendix B, the welfare criterion is derived by a second-order approximation of the average utility of a household around the deterministic long-run steady state (see also Erceg et al., 2000; Gali and Monacelli, 2008; and Woodford, 2003). The welfare function can be written as follows

$$\mathbb{W}_0 = E_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{(1-v\varphi)}{2} \sum_{t=0}^{\infty} \beta^t \left[ (1+\hat{Y}_t)^2 - (\hat{Y}_t - \hat{\zeta}_t)^2 \right] - v\varphi \cdot \frac{\epsilon}{2\kappa} \sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2.$$
(28)

We characterize the welfare implications of the different fiscal policy regimes by means of numerical analysis for four types of shocks, namely shocks to consumer preferences, shocks to the price mark-up, transitory technology shocks and fiscal spending shocks. For the baseline calibration, more than 94% of the welfare losses are driven by these shocks. Measurement errors are only of minor importance in terms of explaining welfare losses. This is due to the fact that fiscal policy only steers about a quarter of GDP and, thus, makes the unsystematic and non-rule based component resulting from spending shocks less important. Nevertheless, in the following, we will take these measurement errors into account when calculating the overall welfare losses given by different rules.

For the structural shocks in our model, Figures 7 to 10 portray the adjustment paths of the annualized inflation rate in the upper panel for the different fiscal policy regimes under consideration. In the lower panel, the response of fiscal authorities under the different regimes is shown. As a reference point, we additionally report how a discretionary optimizing fiscal authority that responds to the predetermined state variables  $shock_t = \tilde{\epsilon}_{j,t}$  with  $j \in \{\zeta, A, \epsilon, v, z, g, \xi, \Psi, Y\}$  and  $b_{t+1}$  behaves by implementing the following rules

$$\hat{G}_t = \phi_1^{opt} shock_{t-1} + \phi_2^{opt} b_t,$$
(29)

where the coefficients  $\phi_1^{opt}$  and  $\phi_2^{opt}$  are chosen such that the welfare loss function, equation (28), is minimized. In order to give a fair comparison, we assume informational symmetry. The optimizing fiscal policymaker can react to the state variables with one period delay such that public expenditures are predetermined in the first quarter across all considered regimes. The following remarks summarize the main findings.

Table 3 Optimal feedback rules

Shock	$\phi_1^{opt}$	$\phi_2^{opt}$
$\tilde{\epsilon}_{\zeta,t}$	-15.56	-0.38
$\tilde{\epsilon}_{\epsilon,t}$	-48.41	-0.36
$\tilde{\epsilon}_{A,t}$	7.50	-0.33
$\tilde{\epsilon}_{g,t}$	-7.22	-0.95
$\tilde{\epsilon}_{\Psi,t}$	-7.40	-0.44
$\tilde{\epsilon}_{Y,t}$	-6.95	-0.47

*Remark 1* All proposed simple fiscal policy regimes perform significantly worse than an optimal discretionary fiscal policymaker that implements rules (29). On average, the welfare loss when following optimal discretion could be reduced by about 79%.

The impulse responses illustrate that an optimal discretionary fiscal policymaker designs a negative correlation between the inflation rate and government expenditures. Such a contractionary policy stance is welfare enhancing as fiscal authorities succeed in favorably influencing wage dynamics and marginal costs by manipulating production plans and, thus, the spending behavior of non-Ricardian households. Accordingly, any policy measure which contributes to inflation stabilization increases welfare. While optimal policy is a good benchmark for comparison from a theoretical perspective, we believe that it is not implementable in practice. Hence, it is worthwhile comparing the welfare consequences of implementable fiscal rule.

Remark 2 In particular, the balanced budget regime and the SGP move government expenditures procyclically to inflation which aggravates the adverse welfare effects of price dispersion as it promotes a boom in overall consumption and provides a boost to inflation.

In the presence of the BB regime and the SGP, the latter as it is estimated by Gali and Perotti (2003) for Germany, government spending, in principle, moves procyclically with inflation, whereas the optimal response would be to move expenditures in the opposite direction. An exception is the presence of a cost-push shock. In this case, tax revenues fall, which implies a fall in government spending when adapting the balanced budget and the SGP (while the other rules imply a rather fixed spending path, see Figure 8). Note, however, that this is the only type of shock in which the balanced budget and the SGP move government spending in the right direction.

Remark 3 The debt brake and the automatic stabilizer generally keep government spending stable and, thus, avoid being a source of economic disturbance. They fare a lot better than the balanced budget rule or the SGP. Switching from a balanced budget rule or the SGP to the pure debt brake regime decreases the welfare loss by 7.23% (2.94%) and 9.90% (13.29%) excluding (including) measurement error, respectively. When applying the debt brake with a higher countercyclical stance, the welfare loss can further be reduced by 2.80% (1.53%) excluding (including) measurement error.

As becomes clear by the description in section 3.2, government spending is kept more or less constant according to the debt brake and the automatic stabilizer. Hence, the inflation dynamics are quite similar. Inspection of Figure 7 shows that, for a consumer preference shock, inflation dynamics are, on impact, a little lower for the DB than for the AS, while the opposite holds for the cost-push shock.

Comparing the results for a cost-push shock, we observe that the automatic stabilizer fares better than the debt brake. This can be explained as follows:



Fig. 9 Technology shock and welfare

Fig. 10 Fiscal shock and welfare

We observe in the first quarter that consumption over both consumer types drops faster for the case of the automatic stabilizer. Accordingly, we observe a more pronounced cut in real wages, which moderates the increase of the inflation rate and is thus welfare enhancing. Therefore inflation on impact is 10 percent lower than under a debt brake regime.

The economic mechanism which drives the result for the debt brake is the mild but highly persistent increase in government expenditures. For the case of highly correlated shocks, movements in public expenditures lead to significant crowding-out effects. Therefore, the anticipation of a highly persistent cut in government expenditures crowds in consumption as the drop in consumption among non-Ricardian households is only moderate. The crowding-in effect is driven by expectations of higher interest rates along the adjustment path. These crowding in effects retard the drops in GDP and accordingly of wages on impact. Only from period three onward, when the cuts in government expenditures materialize, the impulse responses among the two regimes start to converge. In sum, the anticipation effect of highly correlated government expenditures, which only materialize in later periods, drives the differences in welfare results for a debt brake and an automatic stabilizer regime. As the anticipation of highly correlated government expenditures promotes a more moderate drop in wages, this supports higher inflation rates and is, in turn, welfare reducing.

As we have seen already, the SGP regime ties government spending to trend revenues as the DB or AS regimes do, too. Nevertheless, given that there is no clear rule-based feedback on how spending lapses have to be corrected for, the rule still gives room for a somewhat sloppy fiscal policy stance. According to the estimates by Gali and Perotti (2003), the (German) government has taken advantage of this by generally conducting a prolonged procyclical fiscal policy than a balanced budget regime would do. Therefore, given the lack of a rulebased feedback, it fares clearly worse than the debt brake and the automatic stabilizer regimes.

The presence of trend estimation errors does not alter the results from a qualitative perspective. They only diminish the advantages of the AS regime over the DB and the advantages of both these rules over the BB. The presence of estimation errors, however, improves the performance of the AS and the DB regime compared to the SGP. In the light of noisy estimates of potential output and the level of the output gap, policymakers may potentially be badly advised to follow an overly ambitious discretionary stance.

To see whether the evidence gained on the welfare ranking is robust with respect to the deep parameters we conducted a number of sensitivity analysis. The interested reader is referred to the extended working paper version of the paper (see Mayer and Stähler, 2009). While the relative performance of the debt brake in comparison to the automatic stabilizer remains somewhat constant over a wide range of parameters, the relative performance of a balanced budget and the SGP hinge critically on the concrete parameter constellation. It prevails, in particular, that for an increasing share of non-Ricardian households, these regimes fare poorly and ultimately fail to generate a determinate equilibrium. With an increasing share of non-Ricardian households, monetary authorities lose their leverage on the intertemporal consumption decision of the average household, as documented by Gali et al. (2004). As the SGP and a balanced budget regime generate larger amplitude in real wages, this promotes a boom in consumption for rule-of-thumb consumers. If their share increases, this will offset the drop in consumption of Ricardian households and ultimately destabilize the economy.

## 5 Discussion: The feedback parameter of the adjustment account

The aim of this section is to address the question how strongly the balance of the adjustment account should feed back to government spending. In order to analyze this question within our model, it seems natural to minimize the welfare metric presented in equation (28) with respect to the feedback parameter  $\rho$  dependent on each shock. We find that the feedback should be rather small, around  $\rho \approx 0.05$  as in our baseline-calibration, in order for fiscal policy not to create much fluctuation within the economy. Only for discretionary fiscal policy shocks should the feedback be high and, thus, there should be a sharp correction of the earlier lapses because a positive government spending shock and a negative correction through the adjustment account cancel each other out relatively easily. Similar evidence is reported by Kremer and Stegarescu (2008), who report the optimal speed of adaption for German data.

In our model, any balance on the the adjustment account different from zero has to be corrected for, while real-world debt brake regimes allow for some positive/negative balance until the corrective arm kicks in. Hence, it



Fig. 11 Adjustment account dynamics and feedback parameter

seems interesting to see when our model debt brake hits, for example, the Swiss threshold. The Swiss debt brake implies in particular that when the adjustment account hits -6 percent than future spending needs to be cut back. The Swiss debt brake was established in 2001 at the constitutional level and became binding from 2003 onward; see Bodmer (2003). Therefore, we simulate the model over 500,000 quarters, where we draw the shocks from a multivariate normal distribution with standard deviations as reported in Table 2. As in Switzerland, we introduced a critical threshold of -6% of the adjustment account normalized by steady-state fiscal expenditures and computed relevant statistics.

In the upper panel, figure 11 illustrates that the shape of the kernel density function of the adjustment account is driven by the choice of the adjustment parameter. Given Proposition 1, this does not come as a surprise as the distribution flattens with decreasing values of  $\rho$  and exhibits a near random walk behavior for  $\rho = 0.01$ .

The analysis of the simulation leads to the following findings. First, the unconditional probability that the adjustment account is below -6% decreases along a convex line with an increasing feedback parameter  $\rho$  and drops below 1% for  $\rho = 0.25$ . Second, if the adjustment account passes the threshold values of -6%, the unconditionally expected duration of consecutive violations of the threshold value decreases along a convex line with increasing values of  $\rho$ . For the baseline, the expected duration is well above six years. Third, violations of the threshold value are highly persistent if they occur, but are rare events.

The expected duration between two lapses increases along a convex path for increasing values of  $\rho$ . For the baseline calibration, the expected distance between two lapses is 25 years. We conclude that by choosing  $\rho$  appropriately, the unconditional probability, expected duration as well as the distance between two violations is implicitly determined by the government.

# 6 Conclusion

In this paper, we analyze the effects of government spending rules aiming at stabilizing the economy in a sustainable way. We use a conventional New Keynesian model to implement the idea of a balanced budget rule, the Stability and Growth Pact (SGP), a debt brake and a debt brake with higher countercyclical stance. The debt brake, which is currently in action in Switzerland and soon to be implemented in Germany, is a rule tying government spending to real trend revenues similar to the idea of the SGP. Additionally, cyclical surpluses/deficits and expenditures resulting from discretionary fiscal actions and estimation errors are booked on an adjustment account. The (positive) balance of the account cuts future government spending in order to keep debt at a constant level in the long run. The automatic stabilizer implies a higher countercyclical stance in government spending regarding output deviations, while also implementing the adjustment account just described. The balanced budget demands balanced budgets each period, while the other regimes demand structurally balanced budgets.

We find that, not surprisingly, the balanced budget does not stabilize the economy as it moves directly with (projected) government revenues. This similarly holds for the SGP, at least as it has been conducted according to the estimates of Gali and Perotti (2003), mainly due to the fact that it lacks a clear rule-based feedback of earlier spending lapses to current spending and give the government too much room for a procyclical fiscal stance given estimates provided for Germany. The debt brake and the automatic stabilizer have very similar business cycle effects. However, even though the debt brake and the automatic stabilizer rules are both constructed to generate countercyclical spending behavior, government spending in the debt brake regime is still moderately positively correlated with business cycle fluctuations in GDP, while government spending in the automatic stabilizer regime indeed has a mildly countercyclical stance regarding GDP. The weakening of the countercyclical stance in both regimes can be attributed to the interest payments on outstanding debt and the existence of an adjustment account, which serves to generate a constant level of debt in the long run. For the debt brake regime, this even overcompensates the countercyclical stance regarding the correlation of cyclical movements in government spending and GDP. In terms of welfare, calculated as an average consumer loss function, the debt brake and the automatic stabilizer are very similar and, generally, dominate the balanced budget and the SGP regime. Nevertheless, on an aggregate level, the automatic stabilizer seems to generate slightly smaller welfare losses compared to the debt

brake for our baseline calibration. On a disaggregated level (i.e. analyzing each shock separately), the result also holds in principle. An exception is a cost-push shock, where the balanced budget dominates the debt brake. The reason is that a cost-push shock yields higher inflation and lower tax revenue. Lower revenues imply less government spending and, thus, additionally lower aggregate demand and decrease the inflationary pressure under the balanced budget and also the SGP. As inflation is the driving force of welfare losses in this class of models, the balanced budget regime may be the preferable rule for a cost-push shock – even though still contributing to more cyclical fluctuations.

Overall, we find that all rules perform worse than optimal discretionary fiscal actions. However, we believe that the latter are not implementable due to reasons revealed in the political economic literature. Our general finding on an aggregate level is that a rule which steers fiscal expenditures along the trend path and abstains from activism is preferable as it at least prevents to fluctuations being actively introduced into the economy and, thus, acts as an automatic stabilizer. Our model simulation suggests that, in terms of welfare, the pure debt brake regime can be improved by a stronger countercyclical stance.

Unfortunately, trend revenues are not known in practise and have to be estimated. Given that the debt brake regimes and the SGP tie government spending to those trend revenues, it is very likely that this spending component is subject to significant measurement errors. Allowing for such estimation errors in trend in our model, we find that the welfare ranking is not changed even though the welfare gains resulting from the debt brake and the debt brake with a higher countercyclical stance are diminished.

Regarding the design of the debt brake regimes, we can keep hold of the fact that, generally, attention should be devoted to the feedback of the adjustment account to real government spending, which shapes the distribution of the adjustment account. This feedback should be relatively strong for discretionary spending shocks only while adjustment of debt due to other economic shocks should die out more slowly. Generally, we find that by setting up an adjustment account, it is possible to balance the desire to keep debt bounded, while not aggravating the economy at large, especially if fiscal authorities fall prey to measurement errors. The main improvement of the debt brake regimes over, for example, the SGP is thus the rule-based feedback from earlier spending lapses to current government spending because this hampers the policymakers' desire for procyclical spending. Therefore, it is important not to soften this rule-based feedback nor to have policymakers influence trend estimation too much.

# Appendix

## A How to set the feedback of the adjustment account

Assuming that the fundamental shocks (technology shocks (TS), shocks to consumer preferences (CPS), price mark-up shocks (PMS), monetary shocks (MS) and fiscal spending shocks (FS)) are orthogonal as standard in the literature, we can decompose the welfare loss function as a linear combination of the structural shocks, i.e  $\mathbb{W}_0(\rho) = \mathbb{W}_0^{TS}(\rho) + \mathbb{W}_0^{CPS}(\rho) + \mathbb{W}_0^{CPS}(\rho)$  $\mathbb{W}_{0}^{PMS}(\rho) + \mathbb{W}_{0}^{MS}(\rho) + \mathbb{W}_{0}^{FS}(\rho)$ , where all parameters are fixed at their baseline value except  $\rho$ . Then, we continue by calculating  $\mathbb{W}_0(\rho)$ , where  $\rho$  is defined over the following tuple [0.00, 0.05, 0.10, 0.15, 0.20]. While conducting this exercise we find that the welfare loss metric  $\mathbb{W}_0(\rho)$  takes its lowest value for  $\rho = 0.05$ , which we take as our baseline value. A more sophisticated approach would be to find the globally optimal value for each fundamental shock by, for example, the MATLAB routine fmincon, which finds a constrained minimum of a scalar function, starting from an initial estimate. In Figure 12, we report the outcome of such an exercise graphically. It suggests that, if  $\rho$  could be fine-tuned towards a specific shock, the value optimally differed with the shock. If movements in the adjustment account can be traced back to technology or price mark-up shocks, fiscal authorities are well advised not to correct fiscal expenditures to sharply in the following period. For the case of fiscal, monetary and consumer preference shocks, the recommendation is somewhat reversed. If fiscal authorities are the source of economic disturbance, the welfare metric reports evidence that a sharper correction in the following period is appropriate as the relative damage imposed on the consumer can be reduced by a factor of four compared to the case in which fiscal authorities only moderately respond to past lapses in expenditures. For the case of consumer preference and monetary shocks, the welfare metric can be reduced by 10% if fiscal authorities move from a very low feedback ( $\rho = 0.01$ ) to a somewhat higher feedback  $(\rho = 0.05)$ . To be in line with debt brakes actually implemented in Switzerland or which are planed to be implemented in Germany, we assume that the government has no technology at hand to find-tune the response of the adjustment account towards the specific shock and thus set to  $\rho = 0.05$  for all shocks, which is – on average – the best response to movements in the adjustment account.

## **B** Welfare approximation

We know that per-period utility of household j of type i is given by

$$\left\{ \underbrace{\zeta_t \left[ (1-\chi) log\left(C_t^i(j)\right) + \chi log(G_t) \right]}_{=u^i} + \underbrace{\zeta_t \upsilon_t log\left(L_t^i(j)\right)}_{=V^i} \right\}, \tag{30}$$

where i = o, r (see also equation (6)). In what follows, we will derive the second-order Taylor approximation of the consumption part of this equation (indicated by  $u^i$ ) and the leisure



Fig. 12 Optimal feedback coefficient for each shock

part (indicated by  $V^i$ ) separately for convenience. For consumption, we then get

$$\begin{aligned} u_{t}^{i} &\approx \bar{u}^{i} + \bar{u}_{C^{i}}^{i} \left( C_{t}^{i} - \bar{C}^{i} \right) + \frac{1}{2} \bar{u}_{C^{i}C^{i}}^{i} \left( C_{t}^{i} - \bar{C}^{i} \right)^{2} + \bar{u}_{G}^{i} \left( G_{t} - \bar{G} \right) + \frac{1}{2} \bar{u}_{GG}^{i} \left( G_{t} - \bar{G} \right)^{2} \\ &+ \bar{u}_{C^{i}\zeta}^{i} \left( C_{t}^{i} - \bar{C}^{i} \right) \left( \zeta_{t} - \bar{\zeta} \right) + \bar{u}_{G\zeta}^{i} \left( G_{t} - \bar{G} \right) \left( \zeta_{t} - \bar{\zeta} \right) \\ &= \bar{u}^{i} + (1 - \chi) \frac{(C_{t}^{i} - \bar{C}^{i})}{\bar{C}^{i}} - (1 - \chi) \frac{1}{2} \frac{(C_{t}^{i} - \bar{C}^{i})^{2}}{(\bar{C}^{i})^{2}} + \chi \frac{(G_{t} - \bar{G})}{\bar{G}} - \chi \frac{1}{2} \frac{(G_{t} - \bar{G})^{2}}{\bar{G}^{2}} \\ &+ \frac{(\zeta_{t} - \bar{\zeta})}{\bar{\zeta}} \left[ (1 + \chi) \frac{(C_{t}^{i} - \bar{C}^{i})}{\bar{C}^{i}} + \chi \frac{(G_{t} - \bar{G})}{\bar{G}} \right] \\ &= \bar{u}^{i} + (1 + \hat{\zeta}_{t}) \left\{ (1 - \chi) \left[ \frac{\hat{C}_{t}^{i}}{\gamma_{i}} - \frac{1}{2} \frac{(\hat{C}_{t}^{i})^{2}}{\gamma_{i}^{2}} + \frac{1}{2} \frac{(\hat{C}_{t}^{i})^{2}}{\gamma_{i}^{2}} \right] + \chi \left[ \hat{G}_{t} - \frac{1}{2} (\hat{G}_{t})^{2} + \frac{1}{2} (\hat{G}_{t})^{2} \right] \right\} \\ &= \bar{u}^{i} + (1 + \hat{\zeta}_{t}) \left\{ (1 - \chi) \frac{\hat{C}_{t}^{i}}{\gamma_{i}} + \chi \hat{G}_{t} \right\}, \end{aligned}$$

$$(31)$$

where we have used the fact that we defined  $\hat{C}_t^i = \frac{(C_t^i - \bar{C}^i)}{\bar{C}}$  earlier, used the definitions for  $\gamma_r = \frac{v}{1-\chi+v} \frac{1}{1-\bar{N}}$  and  $\gamma_o = \frac{1-\gamma_r \lambda}{1-\lambda}$ , respectively, and made use of the commonly known fact that, for any variable X, it holds that  $(X_t - \bar{X}) \approx \bar{X}[\hat{X}_t + \frac{1}{2}\hat{X}_t^2]$  and  $(X_t - \bar{X})^2 \approx \frac{1}{2}\hat{X}_t^2$  when approximating second order. Furthermore, we have neglected the individual house-hold parameter j for notational convenience and remembered that  $\bar{\zeta} = 1$ . In an analogous

proceeding as before, for the disutility of labor (utility of leisure) this yields

$$\begin{split} V_t^i &\approx \bar{V}^i + \bar{V}_{L^i}^i \left( L_t^i - \bar{L}^i \right) + \frac{1}{2} \bar{V}_{L^i L^i}^i \left( L_t^i - \bar{L}^i \right)^2 + \bar{V}_{L^i \zeta}^i \left( L_t^i - \bar{L}^i \right) \left( \zeta_t - \bar{\zeta} \right) \\ &= \bar{V}^i + v \frac{\left( L_t^i - \bar{L}^i \right)}{\bar{L}^i} - v \frac{1}{2} \frac{\left( L_t^i - \bar{L}^i \right)^2}{(\bar{L}^i)^2} + v \frac{\left( L_t^i - \bar{L}^i \right) \left( \zeta_t - 1 \right)}{\bar{L}^i} \\ &= \bar{V}^i + v \left\{ \left[ \frac{\hat{L}_t^i}{\gamma_i} - \frac{1}{2} \frac{\left( \hat{L}_t^i \right)^2}{\gamma_i^2} + \frac{1}{2} \frac{\left( \hat{L}_t^i \right)^2}{\gamma_i} \right] \right\} + \frac{\hat{L}_t^i}{\gamma_i} \left( v \hat{\zeta}_t \right) \\ &= \bar{V}^i + (1 + \hat{\zeta}_t) v \frac{\hat{L}_t^i}{\gamma_i} = \bar{V}^i + (1 + \hat{\zeta}_t) v \frac{\hat{L}_t^i}{\gamma_i}. \end{split}$$
(32)

Combining the utility of consumption and disutility of labor, we get for household j of type i=o,r that

$$U_{t}^{i}(j) = \underbrace{\bar{u}^{i}(j) + \bar{V}^{i}(j)}_{\bar{U}^{i}} + (1 + \hat{\zeta}_{t}) \left\{ (1 - \chi) \frac{\hat{C}_{t}^{i}(j)}{\gamma_{i}} + \chi \hat{G}_{t} \right\} + (1 + \hat{\zeta}_{t}) v \frac{\hat{L}_{t}^{i}(j)}{\gamma_{i}}.$$
 (33)

Noting that individuals of type r have a constant consumption pattern due to constant labor supply (see equations (12) and (13)), we know that  $\hat{C}_t^r(j) = \hat{C}_t^r$ , where the latter is given by  $\hat{C}_t^r = \gamma_r \hat{C}_t + \varphi \gamma_r \hat{N}_t$ . Due to the assumption of complete markets and state-contingent claims that can be purchased by households of type o, we know that  $\hat{C}_t^o(j) = \hat{C}_t^o$  (see Woodford, 2003, chapter 2 for more details), where the latter is given by  $\hat{C}_t^o = \gamma_o \hat{C}_t - \frac{\lambda \gamma_r \varphi}{1-\lambda} \hat{N}_t$ . Unfortunately, this does not hold for the labor supply (i.e. leisure) except for households of type r. We will come back to this in a second. As we further know that a share  $\lambda$  of households is of type r, while the remainder, i.e.  $(1 - \lambda)$ , is of type o, aggregate per-period utility can be expressed through the second-order Taylor approximation

$$U_{t} = \underbrace{\lambda \bar{U}^{r} + (1-\lambda)\bar{U}^{o}}_{=\bar{U}} + (1+\hat{\zeta}_{t}) \left\{ (1-\chi) \left[ \lambda \frac{\hat{C}_{t}^{r}}{\gamma_{r}} + (1-\lambda) \frac{\hat{C}_{t}^{o}}{\gamma_{o}} \right] + \chi \hat{G}_{t} \right\}$$
$$+ (1+\hat{\zeta}_{t}) \upsilon \left[ \lambda \frac{\frac{1}{\lambda} \int_{0}^{\lambda} \hat{L}_{t}^{r}(j) dj}{\gamma_{r}} + (1-\lambda) \frac{\frac{1}{(1-\lambda)} \int_{\lambda}^{1} \hat{L}_{t}^{o}(j) dj}{\gamma_{o}} \right]$$
(34)

We can use the definition of the consumption aggregate and the labor aggregate, where it holds that

$$\hat{L}_t = \lambda \frac{1}{\gamma_r} \hat{L}_t^r + (1-\lambda) \frac{1}{\gamma_o} \hat{L}_t^o \text{ and } \hat{C}_t = \lambda \frac{1}{\gamma_r} \hat{C}_t^r + (1-\lambda) \frac{1}{\gamma_o} \hat{C}_t^o,$$

where  $\hat{L}_t^i$  and  $\hat{C}_t^i$  denote the per capita log-deviations in the respective household segment. By definition, we know that  $C_t^r = \frac{1}{\lambda} \int_0^{\lambda} C_t^r(j) dj$  and, henceforth,  $\frac{1}{\gamma_r} \hat{C}_t^r = \frac{1}{\gamma_r} \frac{1}{\lambda} \int_0^{\lambda} \hat{C}_t^r(j) dj$ . In complete analogy, we get  $\frac{1}{\gamma_r} \hat{L}_t^r = \frac{1}{\gamma_r} \frac{1}{\lambda} \int_0^{\lambda} \hat{L}_t^r(j) dj$ ,  $\frac{1}{\gamma_o} \hat{C}_t^o = \frac{1}{\gamma_o} \frac{1}{(1-\lambda)} \int_{\lambda}^{1} \hat{C}_t^o(j) dj$  and  $\frac{1}{\gamma_o} \hat{L}_t^o = \frac{1}{\gamma_o} \frac{1}{(1-\lambda)} \int_{\lambda}^{1} \hat{L}_t^o(j) dj$ . Substitution into equation (34) and rearranging gives

$$U_{t} = \bar{U} + (1 + \hat{\zeta}_{t}) \left[ (1 - \chi)\hat{C}_{t} + \chi\hat{G}_{t} \right] - (1 + \hat{\zeta}_{t})\upsilon\varphi\hat{N}_{t},$$

where we have substituted leisure for labor through  $\hat{L}_t = -\frac{\bar{N}}{L}\hat{N}_t = -\frac{\bar{N}}{1-N}\hat{N}_t = -\varphi\hat{N}_t$ . Further, we can use

$$N_t = \int_0^1 N_t(j) dj = \int_0^1 \frac{Y_t(j)}{A_t} dj = \frac{Y_t}{A_t} \int_0^1 \frac{Y_t(j)}{Y_t} dj = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj$$

and log-linearize, which yields  $\hat{N}_t = \hat{Y}_t - \hat{A}_t + \hat{q}_t$ , where  $\hat{q}_t = \log\left(\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj\right)$ . Using standard results as in Woodford (2003), we know that  $q_t = (\epsilon/2) \sigma_t^2$ , where  $\sigma_t^2 = \int_0^1 [p_t(j) - p_t]^2 dj$ , in which the lower case letters p denote second-order log deviations. Substituting into the latest equation for the second-order Taylor approximation, we get

$$U_t = \bar{U} + (1 + \hat{\zeta}_t) \left[ (1 - \chi)\hat{C}_t + \chi \hat{G}_t \right] - (1 + \hat{\zeta}_t) \upsilon \varphi \left[ \hat{Y}_t - \hat{A}_t + \frac{\epsilon}{2} \sigma_t^2 \right]$$

which can be simplified to

$$U_{t} = \bar{U} + (1 + \hat{\zeta}_{t}) \left[ (1 - \chi)\hat{C}_{t} + \chi\hat{G}_{t} \right] - (1 + \hat{\zeta}_{t})\upsilon\varphi\hat{Y}_{t} - \upsilon\varphi\frac{\epsilon}{2}\sigma_{t}^{2} + o\left(||a^{3}||\right) + t.i.p., \quad (35)$$

where terms of order three (such as  $\sigma_t^2 \zeta_t^2$ ) are collected in  $o(||a^3||)$ , while terms independent of policy (such as  $(1 + \hat{\zeta}_t) v \varphi \hat{A}_t$ ) have been put into *t.i.p.*. Using the income identity  $\hat{Y}_t = \gamma_C \hat{C}_t + \gamma_G \hat{G}_t$  and the fact that  $\chi = \gamma_G = (1 - \gamma_C)$  in the efficient steady state, we get

$$U_t = \bar{U} + [1 - v\varphi] \left(1 + \hat{\zeta}_t\right) \hat{Y}_t - v\varphi \frac{\epsilon}{2} \sigma_t^2 + o\left(||a^3||\right) + t.i.p.,\tag{36}$$

Noting that  $\hat{\zeta}_t \hat{Y}_t = \frac{1}{2} \left[ \hat{\zeta}_t^2 + \hat{Y}_t^2 - (\hat{Y}_t - \hat{\zeta}_t)^2 \right]$ , we are able to rearrange this to

$$U_t = \frac{(1 - v\varphi)}{2} \left[ (1 + \hat{Y}_t)^2 - (\hat{Y}_t - \hat{\zeta}_t)^2 \right] - v\varphi \frac{\epsilon}{2} \sigma_t^2 + o\left( ||a^3|| \right) + \overline{t.i.p.},$$
(37)

where

$$\overline{t.i.p.} = t.i.p. + \hat{\zeta}_t^2 \frac{(1-v\varphi)}{2} - \frac{(1-v\varphi)}{2}$$

is the full set of variables independent of policy. Noting that  $\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2 = \frac{\epsilon}{\kappa}\sum_{t=0}^{\infty}\beta^t\hat{\pi}_t^2$  (see Woodford, 2003) and taking conditional expectations at date zero and neglecting al terms higher than second order, the discounted sum of utility streams can be written as equation (28).

#### - Following Appendices not for publication -

### C Log-linearized presentation of the model

In this section, we summarize the model by taking a log-linear approximation of the key equations around a symmetric equilibrium steady state.

Firms From the firm sector, we find that marginal cost are equal to

and, from the production technology, equation (4), we know that

$$\hat{mc}_t(i) = -\hat{A}_t + \hat{w}_t, \tag{38}$$

 $\hat{N}_t = \hat{Y}_t - \hat{A}_t.$ (39)Solving the firm's optimality condition for the optimal reset price and following Gali et al.

$$\hat{\pi}_t = \beta \cdot E_t \left\{ \hat{\pi}_{t+1} \right\} + \kappa \cdot \hat{mc}_t + \hat{\epsilon}_t, \tag{40}$$

 $\hat{\pi}_t = \beta \cdot E_t \left\{ \hat{\pi}_{t+1} \right\} + \kappa \cdot \hat{mc}_t + \hat{\epsilon}_t,$  where  $\kappa = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$  and we defined  $\hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1}.$ 

(2001), we can derive the Phillips curve:

Households The aggregate consumption Euler equation reads:

$$\hat{C}_{t} = E_{t}\hat{C}_{t+1} - \Theta_{n}E_{t}\Delta\hat{N}_{t+1} + \iota^{C}E_{t}\Delta\hat{\tau}_{t+1}^{C} - E_{t}[\hat{R}_{t} - \hat{\pi}_{t+1}] + E_{t}[\hat{\zeta}_{t} - \hat{\zeta}_{t+1}], \quad (41)$$

where  $\Theta_n = \frac{\lambda \gamma_r \varphi}{(1 - \gamma_r \lambda)}$ ,  $\iota^C \equiv \frac{\bar{\tau}^C}{(1 + \bar{\tau}^C)}$ ,  $\varphi = \frac{\bar{N}}{1 - \bar{N}}$ ,  $\gamma_r = \frac{v}{1 - \chi + v} \frac{1}{1 - \bar{N}}$ , and we used that  $\bar{R} = \beta^{-1}$  in steady state. The labor supply schedule is given by

$$\hat{x}_t = \hat{C}_t + \varphi \hat{N}_t + \iota^d \hat{\tau}_t^d + \iota^C \hat{\tau}_t^C, \qquad (42)$$

where  $\iota^d \equiv \frac{\bar{\tau}^d}{(1-\bar{\tau}^d)}$ .

Fiscal authorities The budget constraint, equation (17) reads in log-linearized terms

$$b_{t+1} - \beta^{-1}b_t = \gamma_G \underbrace{\left[\hat{G}_t - (\hat{\Psi}_t - \hat{P}_t)\right]}_{=Primary \ deficit} + \underbrace{\bar{b}}_{<0} \underbrace{\left[(1 - \beta^{-1})\right]}_{<0} \left[\hat{\Psi}_t - \hat{P}_t\right] + \underbrace{\bar{b}}_{\beta\beta}\beta^{-1} \left[\hat{R}_{t-1} - \hat{\pi}_t\right]. \tag{43}$$

Real government revenues evolve according to

$$\hat{\Psi}_t - \hat{P}_t = \underbrace{\frac{\bar{\tau}^d \bar{W} \bar{N}}{\bar{\Psi}}}_{=Rev^L} \left( \hat{\tau}_t^d + \hat{w}_t + \hat{N}_t \right) + \underbrace{\frac{\bar{\tau}^C \bar{P} \bar{C}}{\bar{\Psi}}}_{=Rev^{VAT}} \left( \hat{\tau}_t^C + \hat{C}_t \right), \tag{44}$$

where  $Rev^L = \frac{\bar{\tau}^d(\epsilon-1)}{\epsilon(1-\tau_n^s)[\gamma_G - (1-\beta^{-1})\tilde{b}]}$  and  $Rev^{VAT} = \frac{\gamma_C \bar{\tau}^C}{[\gamma_G - (1-\beta^{-1})\tilde{b}]}$  are the revenue shares of labor and of value added taxes which implies:  $Rev^L + Rev^{VAT} = 1$ . Equation (44) thus determines the deviation of government revenue from its steady-state value. Government spending is given by

$$\hat{G}_t = \frac{(1-\beta^{-1})}{\gamma_G} b_t - \frac{\rho}{\gamma_G} \cdot ac_{t-1} + \frac{1}{\gamma_G \bar{P}\bar{Y}} \cdot \nu_t + \frac{\bar{\tilde{b}}(1-\beta^{-1})}{\gamma_G} \hat{\pi}_t - \beta^{-1} \frac{\bar{\tilde{b}}}{\gamma_G} \hat{R}_{t-1}$$
(45)

$$+\frac{\gamma_G - (1-\beta^{-1})\tilde{\tilde{b}}}{\gamma_G} \left[ \underbrace{\phi_1 \cdot \underbrace{E_{t-1}\left\{\hat{\Psi}_t - \hat{P}_t\right\}}_{BB} - \alpha \cdot \left(\underbrace{E_{t-1}\left\{\hat{Y}_t\right\} + \hat{Z}_{Y,t}}_{AS}\right)}_{=0 \ for DB} + (1-\phi_1)\hat{Z}_{\Psi,t} \right]$$

while the log-linearized adjustment account can be written as

$$ac_{t} = (1-\rho)ac_{t-1} + \frac{\nu_{t}}{\bar{P}\bar{Y}} + \left(\gamma_{G} - (1-\beta^{-1})\bar{\tilde{b}}\right) \left[ \left( \phi_{1} \cdot E_{t-1} \left\{ \hat{\Psi}_{t} \right\} - \varrho \hat{\Psi}_{t} \right) - \left( \phi_{1} \cdot E_{t-1} \left\{ \hat{P}_{t} \right\} - \varrho \hat{P}_{t} \right) - \alpha \left( E_{t-1} \left\{ \hat{Y}_{t} \right\} + \hat{Z}_{Y,t} \right) + (1-\phi_{1})\hat{Z}_{\Psi,t} \right], \quad (46)$$

where  $\phi_1 = \alpha = 0$  and  $\rho = 1$  for the debt brake,  $\phi_1 = \rho = 0$  and  $\alpha > 0$  for the automatic stabilizer and  $\phi_1 = \rho = 1$  and  $\alpha = 0$  for the balanced budget.

*Proof of Proposition 1.* Define a linear combination of variables as given by equation (46) expressed in shock terms. By backward induction, it holds that

$$\begin{aligned} ac_t &= (1-\rho)^{\infty} ac_{t-\infty}^{\infty} + \sum_{k=0}^{\infty} (1-\rho)^k \frac{\nu_{t-k}}{\bar{P}\bar{Y}} + \phi_2 \left(\gamma_G - (1-\beta^{-1})\bar{\tilde{b}}\right) \sum_{k=0}^{\infty} (1-\rho)^k \epsilon_{DB,t-k} \\ &+ \phi_3 \alpha \left(\gamma_G - (1-\beta^{-1})\bar{\tilde{b}}\right) \sum_{k=0}^{\infty} (1-\rho)^k \epsilon_{AS,t-k} \\ &+ \phi_4 \left(\gamma_G - (1-\beta^{-1})\bar{\tilde{b}}\right) \sum_{k=0}^{\infty} (1-\rho)^k \epsilon_{BB,t-k}, \end{aligned}$$

where  $\epsilon_{DB,t} = -\left\{\hat{\Psi}_t - \hat{P}_t\right\}$ ,  $\epsilon_{AS,t} = E_{t-1}\left\{\hat{Y}_t\right\}$  and  $\epsilon_{BB,t} = E_{t-1}\left\{\hat{\Psi}_t - \hat{P}_t\right\} - \left\{\hat{\Psi}_t - \hat{P}_t\right\}$ are white noise processes with  $\phi_2 = 1$ ,  $\phi_3 = \phi_4 = 0$  for the DB,  $\phi_3 = 1$ ,  $\phi_2 = \phi_4 = 0$  for the automatic stabilizer and  $\phi_4 = 1$ ,  $\phi_2 = \phi_3 = 0$  for the balanced budget. It holds that  $ac_t$  will be stationary if  $0 < |\rho| < 1$ , as all sums are bounded.

Monetary authorities Monetary policy follows a Taylor rule

$$\hat{R}_{t} = (1-\mu) \left[ \phi_{\pi} \hat{\pi}_{t} + \phi_{Y} \hat{Y}_{t} \right] + \mu \hat{R}_{t-1} + z_{t},$$
(47)

where  $\phi_{\pi}$  and  $\phi_{Y}$  denote the reaction coefficients towards inflation and output deviations, respectively.  $\mu$  denotes the degree of interest rate smoothing.  $z_t$  defines the monetary shock.

Market clearing Market clearing implies that

$$\hat{Y}_t = \gamma_C \hat{C}_t + \gamma_G \hat{G}_t, \tag{48}$$

where  $\gamma_C$  and  $\gamma_G$  are the shares of output devoted to private and public consumption and can be expressed in terms of deep parameters.

**Shocks** For shocks, we assume:  $\zeta_t = \rho_{\zeta} \cdot \zeta_{t-1} + \tilde{\zeta}_t$ ,  $A_t = \rho_A \cdot A_{t-1} + \tilde{A}_t$ ,  $\epsilon_t = \rho_\epsilon \cdot \epsilon_{t-1} + \tilde{\epsilon}_t$ ,  $\upsilon_t = \rho_{\upsilon} \cdot \upsilon_{t-1} + \tilde{\upsilon}_t$ ,  $z_t = \rho_z \cdot z_{t-1} + \tilde{z}_t$ ,  $\upsilon_t = \rho_{\upsilon} \cdot \upsilon_{t-1} + \tilde{\nu}_t$  and  $\xi_t = \rho_{\xi} \cdot \xi_{t-1} + \tilde{\xi}_t$ , where  $\tilde{\zeta}_t, \tilde{A}_t, \tilde{\epsilon}_t, \tilde{\upsilon}_t, \tilde{z}_t, \tilde{\upsilon}_t$  and  $\tilde{\xi}_t$  are random i.i.d. shocks. Hence, equations (38) to (48), as well as the shock rules, describe the economy.

#### D Aggregation of household sector

Households' FOCs: The first-order conditions for optimizing households are

$$\frac{\partial(.)}{\partial C_t^o(j)} = \frac{(1-\chi)\zeta_t}{C_t^o(j)} - \lambda_t^o(1+\tau_t^C) = 0,$$
(49)

$$\frac{\partial(.)}{\partial L_t^o(j)} = \frac{v\zeta_t}{L_t^o(j)} - \lambda_t^o(1 - \tau_t^d)w_t = 0,$$
(50)

and

$$\frac{\partial(.)}{\partial B^o_{t+1}(j)} = -\frac{1}{P_t}\lambda^o_t + \beta E_t \left\{\lambda^o_{t+1}\frac{R_t}{P_{t+1}}\right\} = 0, \tag{51}$$

where  $\lambda_t^o$  is the Lagrangian multiplier associated with the budget constraint, equation (7). From equation (51), we know that

$$R_t^{-1} = \beta E_t \left\{ \frac{\lambda_{1,t+1}^o}{\lambda_{1,t}^o} \frac{P_t}{P_{t+1}} \right\},$$
(52)

which is the stochastic discount factor. Using equation (49) yields equations (8) and (9). The first-order conditions for rule-of-thumb consumers are given by

$$\frac{\partial(.)}{\partial C_t^r(j)} = \frac{(1-\chi)\zeta_t}{C_t^r(j)} - \lambda_t^r(1+\tau_t^C) = 0$$
(53)

and

$$\frac{\partial(.)}{\partial L_t^r(j)} = \frac{\upsilon \zeta_t}{L_t^r(j)} - \lambda_t^r (1 - \tau_t^d) w_t = 0, \tag{54}$$

where  $\lambda_t^r$  is the Lagrangian multiplier associated with the corresponding budget constraint. From equations (53) and (54), we derive equation (11).

Aggregate consumption Euler equation: The aim of the rest of this section is to derive an aggregate consumption Euler equation (in log-linearized terms) expressed only in aggregate variables and deep parameters. To achieve this, we revert to the households' consumption decisions derived in subsections 2.2.1 and 2.2.2. This means that we have to back-step every now and then to simplify the resulting equations. We know with the help of equation (12) that

$$N_t = \lambda N_t^r + (1 - \lambda) N_t^o = \frac{\lambda \cdot (1 - \chi)}{(1 - \chi) + v_t} + (1 - \lambda) N_t^o$$
(55)

and that

1

$$C_{t} = \lambda C_{t}^{r} + (1 - \lambda)C_{t}^{o}$$

$$= \lambda \left[ \frac{(1 - \chi)}{\upsilon} w_{t} \frac{(1 - \tau_{t}^{d})}{(1 + \tau_{t}^{C})} L_{t}^{r} \right] + (1 - \lambda) \left[ \frac{(1 - \chi)}{\upsilon} w_{t} \frac{(1 - \tau_{t}^{d})}{(1 + \tau_{t}^{C})} L_{t}^{o} \right]$$

$$= \left[ \frac{(1 - \chi)}{\upsilon} w_{t} \frac{(1 - \tau_{t}^{d})}{(1 + \tau_{t}^{C})} \right] \underbrace{[\lambda L_{t}^{r} + (1 - \lambda) L_{t}^{o}]}_{\equiv L_{t}}, \tag{56}$$

where the index j has been dropped for notational convenience<sup>3</sup>, while  $C_t^r$  is given by equation (11) and  $C_t^o$  by equation (9). Log-linearization of equation (56) yields

$$\hat{C}_t - \hat{L}_t = \hat{w}_t - \iota^d \hat{\tau}_t^d - \iota^C \hat{\tau}_t^C,$$

where  $\iota^d \equiv \frac{\bar{\tau}^d}{(1-\bar{\tau}^d)}$  and  $\iota^C \equiv \frac{\bar{\tau}^C}{(1+\bar{\tau}^C)}$ . We know that  $\hat{L}_t = -\frac{\bar{N}}{1-\bar{N}}\hat{N}_t = -\varphi\hat{N}_t$  from loglinearizing  $L_t = 1 - N_t$ , where  $\varphi = \frac{\bar{N}}{1-N}$  is the inverse of the Frisch labor supply elasticity. Substituting  $\hat{L}_t$  and rearranging thus gives

$$\hat{w}_t = \hat{C}_t + \varphi \hat{N}_t + \iota^d \hat{\tau}_t^d + \iota^C \hat{\tau}_t^C, \tag{57}$$

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<sup>&</sup>lt;sup>3</sup> Note that, due to state-contingent claims available for optimizing households, which are generally assumed in this type of model, and the fact that rule-of-thumb consumers consume all of their income, each individual household's consumption in i = o, r is equal anyway (see Woodford, 2003, chapter 2).

which is equation (42) of the main text.

We now come to some side-steps to be able to derive the aggregate consumption Euler equation. From equation (13) we know that, in steady state,  $\bar{C}^r = \frac{(1-\bar{\tau}^d)(1-\chi)}{((1-\chi)+\upsilon)(1+\bar{\tau}^C)}\bar{w}$ , while, from equation (56) and  $\bar{L} = 1 - \bar{N}$ , it is clear that  $\bar{C} = (1 - \bar{N})\frac{(1-\bar{\tau}^d)(1-\chi)}{\upsilon(1+\bar{\tau}^C)}\bar{w}$ , which yields

$$\frac{\bar{C}^r}{\bar{C}} = \frac{\upsilon}{1 - \chi + \upsilon} \cdot \frac{1}{1 - \bar{N}} \equiv \gamma_r, \tag{58}$$

where  $\gamma_r$  is, thus, the per capita consumption share of rule-of-thumb households relative to total per capita consumption. As we further know from equation (56) that  $\bar{C} = \lambda \bar{C}^r + (1 - \lambda) \bar{C}^o$ , we find that  $1 = \lambda \frac{\bar{C}^r}{C} + (1 - \lambda) \frac{\bar{C}^o}{C}$ , which, using equation (58) can be reformulated as  $\frac{\bar{C}^o}{C} - \frac{1}{2} - \frac{\lambda}{C} \frac{v}{C} - \frac{1}{2} - \frac{v}{C}$  (50)

$$\frac{C^o}{\bar{C}} = \frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda} \underbrace{\frac{\upsilon}{1-\chi+\upsilon} \frac{1}{1-\bar{N}}}_{=\gamma_T} = \frac{1-\gamma_r \lambda}{1-\lambda} \equiv \gamma_o,$$
(59)

which, equivalently, gives the per capita consumption share of optimizing households relative to total per capita consumption. (Note that, whenever optimizing households consume more than rule-of-thumb households,  $\gamma_o > 1$  may well be possible and vice versa). Using  $\bar{L} = \lambda \bar{L}^r + (1 - \lambda) \bar{L}^o = \lambda (1 - \bar{N}^r) + (1 - \lambda) \bar{L}^o$ , where  $\bar{N}^r$  is given by equation (12), we know that  $\bar{L} = \lambda \left(1 - \frac{(1 - \chi)}{1 - \chi + \upsilon}\right) + (1 - \lambda) \bar{L}^o$ , which, dividing both sides by  $\bar{L} = (1 - \bar{N})$  yields  $1 = \gamma_r \lambda + (1 - \lambda) \frac{\bar{L}^o}{L}$ , where  $\gamma_r$  is given by equation (58). Thus,

$$\frac{\bar{L}^o}{\bar{L}} = \frac{1 - \gamma_r \lambda}{1 - \lambda} = \gamma_o \tag{60}$$

is also the per capita leisure of optimizing households relative to total per capita leisure. From equation (8), we know that, for the optimizing households, it holds that

$$\frac{\zeta_t}{C_t^o \cdot (1 + \tau_t^C)} = \beta R_t E_t \left\{ \frac{\zeta_{t+1}}{C_{t+1}^o \cdot (1 + \tau_{t+1}^C)} \cdot \frac{P_t}{P_{t+1}} \right\}.$$
 (61)

A Taylor expansion and use of  $E_t r_t = E_t \left\{ R_t \cdot \frac{P_t}{P_{t+1}} \right\}$  yields

$$\frac{\bar{\zeta}}{\bar{C}^{o} \cdot (1+\bar{\tau}^{C})} \left[ -\frac{(C_{t}^{o}-\bar{C}^{o})}{\bar{C}} \frac{\bar{C}}{\bar{C}^{o}} + \frac{(\zeta_{t}-\bar{\zeta})}{\bar{\zeta}} - \frac{\bar{\tau}^{C}}{1+\bar{\tau}^{C}} \frac{(\tau_{t}^{C}-\bar{\tau}^{C})}{\bar{\tau}^{C}} \right] \\ = \beta \bar{r} \frac{\bar{\zeta}}{\bar{C}^{o} \cdot (1+\bar{\tau}^{C})} E_{t} \left[ -\frac{(C_{t+1}^{o}-\bar{C}^{o})}{\bar{C}} \frac{\bar{C}}{\bar{C}^{o}} + \frac{(\zeta_{t+1}-\bar{\zeta})}{\bar{\zeta}} - \frac{\bar{\tau}^{C}}{1+\bar{\tau}^{C}} \frac{(\tau_{t+1}^{C}-\bar{\tau}^{C})}{\bar{\tau}^{C}} + \frac{1}{\bar{\tau}} (r_{t}-\bar{r}) \right].$$

We now define  $\hat{C}_t^o \equiv \frac{(C_t^o - \bar{C}^o)}{\bar{C}}$  and  $\hat{L}_t^o \equiv \frac{(L_t^o - \bar{L}^o)}{\bar{L}}$  and note that  $\frac{\bar{C}}{\bar{C}^o} = \frac{\bar{L}}{\bar{L}^o} = \frac{1}{\gamma_o}$  (see equations (59) and (60))<sup>4</sup> as well as  $\bar{r} = \beta^{-1}$ . Substitution and rearranging yields

$$\left[-\hat{C}_t^o\frac{1}{\gamma_o}+\hat{\zeta}_t-\iota^C\hat{\tau}_t^C\right]=E_t\left[-\hat{C}_{t+1}^o\frac{1}{\gamma_o}+\hat{\zeta}_{t+1}-\iota^C\hat{\tau}_{t+1}^C+\hat{r}_t\right],$$

where  $\iota^C = \frac{\bar{\tau}^C}{1 + \bar{\tau}^C}$ . Rearranging gives

$$\hat{C}_{t}^{o} = E_{t}\hat{C}_{t+1}^{o} + \gamma_{o}\left\{ \iota^{C}E_{t}[\hat{\tau}_{t+1}^{C} - \hat{\tau}_{t}^{C}] + E_{t}[\hat{\zeta}_{t} - \hat{\zeta}_{t+1}] - \hat{\tau}_{t} \right\}.$$
(62)

<sup>&</sup>lt;sup>4</sup> Note that this is then the deviation of  $C_t^o$  or  $L_t^o$  from its steady-state value evaluated at the steady-state value of total consumption/leisure. This is corrected by dividing this term by  $\gamma_o$  in the following equation. The slightly different definition from the standard definition will be useful for further calculations.

We define  $\Delta \hat{L}_{t+1}^o = [\hat{L}_{t+1}^o - \hat{L}_t^o]$  and  $\Delta \hat{\tau}_{t+1}^C = [\hat{\tau}_{t+1}^C - \hat{\tau}_t^C]$  for later use. From equation (55), we know that  $\frac{N_t}{N} = \frac{\lambda \cdot (1-\chi)}{N((1-\chi)+\nu)} + (1-\lambda)\frac{N_t^o}{N}$ . A first-order Taylor expansion implies that

$$\hat{N}_t^o = \frac{N_t}{1 - \lambda} \quad \Rightarrow \quad \hat{L}_t^o = -\frac{\varphi}{1 - \lambda} \hat{N}_t \tag{63}$$

because  $\hat{L}_t = -\frac{\bar{N}}{1-\bar{N}}\hat{N}_t = -\varphi\hat{N}_t$ , where we have defined  $\hat{N}_t^o = \frac{N_t^o - \bar{N}^o}{\bar{N}}$ . From equation (13) and (56), it must hold that  $\frac{C_t^r}{C} = \frac{C_t}{C} \frac{v}{(1-\chi)+v} \frac{1}{1-N_t}$ , where a first-order Taylor expansion yields

$$\hat{C}_t^r = \gamma_r \hat{C}_t + \varphi \gamma_r \hat{N}_t, \qquad (64)$$

where we have used the definition for  $\gamma_r$  (see equation (58)),  $\varphi = \bar{N}/(1-\bar{N})$  and defined  $\hat{C}_t^r = \frac{C_t^r - \bar{C}^r}{C}$ . Log-linearizing aggregate consumption  $C_t = \lambda C_t^r + (1 - \lambda)C_t^o$  yields  $\hat{C}_t = \lambda \hat{C}_t^r + (1 - \lambda)\hat{C}_t^o$ . Solving this for  $\hat{C}_t^o$  and using equation (64) yields

$$\hat{C}_{t}^{o} = \underbrace{\frac{1 - \gamma_{r}\lambda}{1 - \lambda}}_{=\gamma_{o}} \hat{C}_{t} - \frac{\lambda\gamma_{r}\varphi}{1 - \lambda}\hat{N}_{t}.$$
(65)

From equation (63), we know that  $\Delta \hat{L}_{t+1}^o = -\frac{\varphi}{(1-\lambda)} \Delta \hat{N}_{t+1}$  must hold. Substituting this and equation (65) into equation (62) we get

$$\gamma_o \hat{C}_t - \frac{\lambda \gamma_r \varphi}{(1-\lambda)} \hat{N}_t - \gamma_o \hat{\zeta}_t = \gamma_o E_t \hat{C}_{t+1} - \frac{\lambda \gamma_r \varphi}{(1-\lambda)} E_t \hat{N}_{t+1} + \gamma_o \left\{ \iota^C E_t \Delta \hat{\tau}_{t+1}^C - E_t [\hat{\zeta}_{t+1}] - E_t [\hat{R}_t - \hat{\pi}_{t+1}] \right\},$$

where we have used  $\hat{r}_t = [\hat{R}_t - \hat{\pi}_{t+1}]$ , with  $\hat{\pi}_{t+1} \approx \hat{P}_{t+1} - \hat{P}_t$ . Dividing by  $\gamma_o$ , i.e. multiplying by  $\frac{(1-\lambda)}{1-\gamma_r\lambda}$ , we get equation (41). Equation (41) is the standard aggregate consumption Euler equation expressed in aggregate variables and deep parameters only. Individual steady-state relations have been substituted out but, of course, still drive equation (41) through the "correct" substitution.

### E The fiscal spending rule

Before deriving the spending rule in log-linearized terms, it seems appropriate to have some steady-state considerations regarding the spending rule, equations (19) and (23), and the adjustment account, equations (20) and (24). From these equations, we see that, in steady state.

$$(\bar{R}-1)\bar{\tilde{b}} + \frac{\bar{G}}{\bar{Y}} = \frac{\bar{\Psi}}{\bar{P}\bar{Y}} - \rho \cdot \bar{a}c \tag{66}$$

and

$$\bar{ac} = (1-\rho)\bar{ac} + \frac{\bar{\Psi}}{\bar{P}\bar{Y}} - \frac{\bar{\Psi}}{\bar{P}\bar{Y}} \Rightarrow \rho \cdot \bar{ac} = 0.$$
(67)

As we know that  $\rho > 0$  if the adjustment account feeds back on government spending,  $\bar{ac} = 0$ has to hold in steady state. Then, from equation (66), we know that  $\frac{\bar{\Psi}}{PY} = \gamma_G - (1 - \beta^{-1})\tilde{b}$ , where we have used the definition  $\gamma_G = \frac{\bar{G}}{\bar{Y}}$  and the fact that  $\bar{R} = \beta^{-1}$  in steady state. Note that these conditions correspond to the evolution of debt in steady state, given by equation (17) in steady state, which also gives  $\frac{\bar{\Psi}}{\bar{PY}} = \gamma_G - (1 - \beta^{-1})\tilde{b}$ , but where the adjustment account has not yet been taken into account. Hence, the fact that  $\bar{ac} = 0$  in steady state is evolution of the product of the steady state is the model. consistent with the model.

A first-order Taylor expansion of equation (17) yields

$$\begin{split} & \underbrace{\left[\tilde{\tilde{b}} + \frac{\bar{\Psi}}{\bar{P}\bar{Y}}\right]}_{=\gamma_G + \beta^{-1}\bar{\tilde{b}}} + \underbrace{(\tilde{b}_{t+1} - \bar{b})}_{=b_{t+1}} + \frac{1}{\bar{P}\bar{Y}} \left(\Psi_t - \bar{\Psi}\right) - \frac{\bar{\Psi}}{\bar{P}^2\bar{Y}} \left(P_t - \bar{P}\right) \\ & = \underbrace{\left[\bar{R}\bar{\tilde{b}} + \frac{\bar{G}}{\bar{Y}}\right]}_{=\gamma_G + \beta^{-1}\bar{\tilde{b}}} + \frac{\bar{b}}{\bar{b}} \left(R_{t-1} - \bar{R}\right) + \bar{R}\underbrace{\left(\tilde{b}_t - \bar{b}\right)}_{=b_t} + \frac{\bar{R}\bar{\tilde{b}}}{\bar{P}} \left(P_{t-1} - \bar{P}\right) \\ & - \frac{\bar{R}\bar{\tilde{b}}\bar{P}}{\bar{P}^2} \left(P_t - \bar{P}\right) + \frac{1}{\bar{Y}} \left(G_t - \bar{G}\right), \end{split}$$

where use has been made of equations (18) and (66) to derive the terms in the under-braces. Using the definition for any variable's deviation around its steady state as well as equation (66) and  $\bar{R} = \beta^{-1}$ , we can rearrange the above equation to  $b_{t+1} + \left[\gamma_G - \tilde{\bar{b}}(1 - \beta^{-1})\right] \left(\hat{\Psi}_t - \hat{P}_t\right) = \beta^{-1}b_t + \beta^{-1}\bar{\bar{b}}\left(\hat{R}_{t-1} + \hat{P}_{t-1} - \hat{P}_t\right) + \gamma_G \hat{G}_t$ .<sup>5</sup> Using the definition  $\hat{\pi}_t \approx \hat{P}_t - \hat{P}_{t-1}$ , rearranging yields equation (43).

A first-order Taylor expansion of the spending rule, equation (23), yields

$$\underbrace{\left[(\bar{R}-1)\bar{b}+\frac{\bar{G}}{\bar{Y}}\right]}_{=\gamma_G-(1-\beta^{-1})\bar{b}} + (\bar{R}-1)\underbrace{(\bar{b}_t-\bar{b})}_{=b_t} + \bar{b}\left(R_{t-1}-\bar{R}\right) + \frac{(\bar{R}-1)\bar{b}}{\bar{P}}\left(P_{t-1}-\bar{P}\right)}_{=b_t}$$
$$-\frac{(\bar{R}-1)\bar{b}\bar{P}}{\bar{P}^2}\left(P_t-\bar{P}\right) + \frac{1}{\bar{Y}}\left(G_t-\bar{G}\right)$$
$$=\underbrace{\left[\frac{\bar{\Psi}}{\bar{P}\bar{Y}}\right]}_{=\gamma_G-(1-\beta^{-1})\bar{b}} - \underbrace{\left[\frac{\bar{\Psi}}{\bar{P}\bar{Y}}\right]}_{=\gamma_G-(1-\beta^{-1})\bar{b}} \cdot \alpha \cdot E_{t-1}\left\{\hat{Y}_t\right\} + \frac{\nu_t}{\bar{P}\bar{Y}} - \rho \cdot ac_{t-1},$$

whereas a first-order Taylor expansion of equation (19) yields

$$\underbrace{\begin{bmatrix} (\bar{R}-1)\bar{b} + \frac{\bar{G}}{\bar{Y}} \end{bmatrix}}_{=\gamma_G - (1-\beta^{-1})\bar{b}} + (\bar{R}-1)\underbrace{(\tilde{b}_t - \bar{b})}_{=b_t} + \bar{b}\left(R_{t-1} - \bar{R}\right) + \frac{(\bar{R}-1)\bar{b}}{\bar{P}}\left(P_{t-1} - \bar{P}\right)}_{=b_t} - \frac{(\bar{R}-1)\bar{b}\bar{P}}{\bar{P}^2}\left(P_t - \bar{P}\right) + \frac{1}{\bar{Y}}\left(G_t - \bar{G}\right) \\ = \underbrace{\begin{bmatrix} \bar{\Psi}\\ \bar{P}\bar{Y} \end{bmatrix}}_{=\gamma_G - (1-\beta^{-1})\bar{b}} + \underbrace{\begin{bmatrix} \bar{\Psi}\\ \bar{P}\bar{Y} \end{bmatrix}}_{=\gamma_G - (1-\beta^{-1})\bar{b}} \cdot E_{t-1}\left\{\hat{\Psi}_t - \hat{P}_t\right\} + \frac{\nu_t}{\bar{P}\bar{Y}} - \rho \cdot ac_{t-1}.$$

where we have already used the fact that  $\bar{ac} = \bar{\nu} = 0$ , the definition of equation (18) and the steady-state condition (66). Solving for  $\hat{G}_t$ , and combining the two previous equations yields equation (45).

<sup>&</sup>lt;sup>5</sup> Remember that  $\bar{\Psi}/(\bar{P}\bar{Y}) = \gamma_G - \bar{\tilde{b}}(1-\beta^{-1}).$ 

A first-order Taylor expansion of equation (24) yields

$$(ac_t - \bar{a}c) = (1 - \rho)(ac_{t-1} - \bar{a}c) + \underbrace{\frac{\bar{a}c}{\bar{P}}(P_{t-1} - \bar{P}) - \frac{\bar{a}c}{\bar{P}^2}(P_t - \bar{P})}_{=0} + \underbrace{\frac{\bar{\nu}t}{\bar{P}\bar{Y}}}_{=0} - \underbrace{\frac{\bar{\Psi}}{\bar{P}\bar{Y}}}_{=\gamma_G - (1 - \beta^{-1})\bar{b}} \cdot \left[\alpha \left(-E_{t-1}\left\{\hat{Y}_t\right\}\right) + \varrho \left(\hat{\Psi}_t - \hat{P}_t\right)\right],$$

while a first-order Taylor expansion of equation (20) is given by

$$(ac_{t} - \bar{a}c) = (1 - \rho)(ac_{t-1} - \bar{a}c) + \underbrace{\frac{\bar{a}c}{\bar{P}}(P_{t-1} - \bar{P}) - \frac{\bar{a}c}{\bar{P}^{2}}(P_{t} - \bar{P})}_{=0} + \underbrace{\frac{\bar{\Psi}}{\bar{P}\bar{Y}}}_{=\gamma_{G} - (1 - \beta^{-1})\bar{\tilde{b}}} \cdot \left[ \left( E_{t-1} \left\{ \hat{\Psi}_{t} \right\} - \hat{\Psi}_{t} \right) - \left( E_{t-1} \left\{ \hat{P}_{t} \right\} - \hat{P}_{t} \right) \right],$$

which can be combined to equation (46).

### F Steady-state considerations and social planner's solution

We know that, in the long run, equilibrium prices will be equal to the flex-price equilibrium. We know then that it holds that

$$\bar{mc} = \frac{\epsilon - 1}{\epsilon},\tag{68}$$

where we have used  $\tilde{P}_t(i) = P_t^{flex}$  which holds in the long-run steady-state. Additionally, we know from the cost minimization problem of a representative firm that

$$\bar{w} = \bar{m}c\frac{\bar{Y}}{\bar{N}}\frac{1}{(1-\tau_n^s)}.$$
(69)

From equation (56), we know that  $\bar{w} = \frac{v}{(1-\chi)} \frac{\bar{C}}{1-N} \frac{(1+\bar{\tau}^C)}{(1-\bar{\tau}^d)}$ , which, in combination with equation (69) yields

$$\frac{(\epsilon - 1)}{\epsilon} \frac{(1 - \bar{\tau}^d)}{(1 + \bar{\tau}^C)} = (1 - \tau_n^s) \frac{\upsilon}{(1 - \chi)} \frac{\bar{C}}{(1 - \bar{N})} \frac{\bar{N}}{\bar{Y}}.$$

As we know that in an undistorted steady state without price mark-up, it must hold that  $1 = \frac{v}{(1-\chi)} \frac{\bar{C}}{(1-\bar{\chi})} \frac{\bar{N}}{\bar{Y}}$ , the following condition for the subsidy  $\tau_n^s$  needs to hold in order to reach the undistorted steady state in our model set-up

$$\tau_n^s = 1 - \frac{(\epsilon - 1)}{\epsilon} \frac{(1 - \bar{\tau}^d)}{(1 + \bar{\tau}^C)}.$$
(70)

With this subsidy at hand, it holds that

$$\frac{\bar{N}}{1-\bar{N}} = \frac{1}{\gamma_C} \frac{(1-\chi)}{\nu},$$
(71)

where we have defined  $\gamma_C = \frac{\bar{C}}{Y}$ . The solution for the steady-state labor supply is thus given by  $\bar{N} = \frac{(1-\chi)}{\chi\gamma_C + (1-\chi)}$ . This implies that  $\bar{N}$  can be expressed in exogenous parameters if we are able to find a solution for  $\gamma_C$  which we will derive now. Note that from steady-state conditions resulting from equation (66), we know that  $\frac{\bar{\Psi}}{PY} = \gamma_G - (1 - \beta^{-1})\bar{\tilde{b}}$  holds, where  $\bar{\tilde{b}} = 0$  in the zero steady-state debt case. Further, it holds that (see equation (16))

$$\frac{\bar{\Psi}}{\bar{P}\bar{Y}} = \bar{\tau}^d \bar{w} \frac{\bar{N}}{\bar{Y}} + \bar{\tau}^C \frac{\bar{C}}{\bar{Y}},$$

where  $\bar{\tau}^L = \bar{\tau}^w + \bar{\tau}^d$ . Using equations (68) and (69), the definition  $\gamma_C = \frac{\bar{C}}{Y}$  and combining the last two equations yields

$$\gamma_G - (1 - \beta^{-1})\overline{\tilde{b}} = \overline{\tau}^L \frac{\epsilon - 1}{\epsilon} \frac{1}{(1 - \tau_n^s)} + \overline{\tau}^C \gamma_C.$$

$$\tag{72}$$

From the resource constraint,  $\overline{Y} = \overline{C} + \overline{G}$ , we know that  $1 = \frac{\overline{C}}{Y} + \frac{\overline{G}}{Y} = \gamma_C + \gamma_G$ . Using this and equation (72), we then find that

$$\frac{\bar{G}}{\bar{Y}} = \gamma_G = \frac{1}{(1+\bar{\tau}^C)} \left\{ (1-\beta^{-1})\bar{\tilde{b}} + \frac{\epsilon-1}{\epsilon}\bar{\tau}^L \frac{1}{(1-\tau_n^s)} + \bar{\tau}^C \right\}$$
(73)

is determined by exogenous parameters. Hence, from the resource constraint, we know that

$$\frac{\bar{C}}{\bar{Y}} \equiv \gamma_C = 1 - \gamma_G. \tag{74}$$

From the first-order condition of the cost minimizing problem of the firm, we know that  $\bar{mc} = \frac{\bar{N}}{Y} [(1 - \tau_n^s)\bar{w}]$ , where  $\frac{\bar{N}}{Y} = \frac{1}{A} = 1$  as  $\bar{A} = 1$  (see equation (4)), which, using equation (68) and rearranging yields

$$\bar{w} = \frac{1}{(1-\tau_n^s)} \frac{\epsilon-1}{\epsilon} = \frac{(1+\bar{\tau}^C)}{(1-\bar{\tau}^d)},\tag{75}$$

where use has been made of equation (70). From equation (56) we can then calculate

$$\bar{C} = \frac{(1-\chi)}{v} \cdot \frac{1-\bar{\tau}^d}{1+\bar{\tau}^C} (1-\bar{N})\bar{w} = \frac{(1-\chi)}{v} \cdot (1-\bar{N}),$$
(76)

where  $\bar{w}$  is given by equation (75) and  $\bar{N}$  by equation (71). Using equation (76) and  $\gamma_C = \frac{\bar{C}}{Y}$ , where  $\gamma_C$  is given by equation (74), we can calculate

$$\bar{Y} = \frac{\bar{C}}{\gamma_C}.$$
(77)

An analogous proceeding allows us – using equations (73) and (77) – to derive

$$\bar{G} = \gamma_G \bar{Y} = \bar{C} \frac{\gamma_G}{\gamma_C}.$$
(78)

Further, using equation (72), we know that

$$1 = \underbrace{\frac{\bar{\tau}^d(\epsilon - 1)}{\epsilon(1 - \tau_n^s)[\gamma_G - (1 - \beta^{-1})\bar{\tilde{b}}]}}_{=Rev^L} + \underbrace{\frac{\bar{\tau}^C \gamma_C}{[\gamma_G - (1 - \beta^{-1})\bar{\tilde{b}}]}}_{=Rev^{Vat}},$$
(79)

where all parameters are known from the calculation above. This implies that we are able to express all aggregated variables in terms of exogenous parameters. Note that these aggregate variables in steady state are independent of the implemented government spending policy regime, i.e. they are independent of whether automatic stabilizers, the debt brake or no restriction on government spending apply. Note further that  $\chi = \gamma_G$  following from an "optimal social planner's solution" (see also Gali and Monacelli, 2008, who apply exactly the same calculation procedure that is necessary here).

**Social planner's solution:** In the following, we will show that the competitive steady state equilibrium we just derived is identical to the solution of the social planner, if  $\gamma_G = \chi$  (which we assume the social planner can choose). Therefore, in the following, we can claim to approximate around an efficient steady state. The optimal allocation of the model can be described by a social planner maximizing

$$SP_{Problem} = \max\left\{\zeta_t\left\{\lambda\left[(1-\chi)log(C_t^r) + \chi log(G_t) + \nu log(L_t^r)\right]\right\} + (1-\lambda)\left[(1-\chi)log(C_t^o) + \chi log(G_t) + \nu log(L_t^o)\right]\right\}\right\}$$
(80)

with respect to  $C_t^r$ ,  $C_t^o$ ,  $L_t^r$ ,  $L_0^r$  and  $G_t$  subject to the constraints  $Y_t = C_t + G_t$  (market clearing),  $Y_t = A_t N_t$  (technology constraint),  $1 = N_t + L_t$  (labor constraint), where  $L_t = \lambda L_t^r + (1 - \lambda) L_t^o$  and  $C_t = \lambda C_t^r + (1 - \lambda) C_t^o$ , which can be summarized in

$$A_t \left[ 1 - (\lambda L_t^r + (1 - \lambda) L_t^o) \right] = \lambda C_t^r + (1 - \lambda) C_t^o + G_t.$$
(81)

The corresponding first-order conditions are given by

$$\frac{\partial(.)}{\partial C_t^r} = \zeta_t \lambda (1-\chi) \frac{1}{C_t^r} - \lambda \cdot o = 0,$$
  
$$\frac{\partial(.)}{\partial C_t^o} = \zeta_t (1-\lambda) (1-\chi) \frac{1}{C_t^o} - (1-\lambda) \cdot o = 0,$$
  
$$\frac{\partial(.)}{\partial L_t^r} = \zeta_t \lambda (1-\chi) \frac{1}{L_t^r} - \lambda \cdot o = 0,$$
  
$$\frac{\partial(.)}{\partial L_t^o} = \zeta_t (1-\lambda) (1-\chi) \frac{1}{L_t^o} - (1-\lambda) \cdot o = 0$$

and

$$\frac{\partial(.)}{\partial G_t} = \zeta_t \frac{\chi}{C_t^r} - o = 0,$$

where o is the corresponding Lagrangian parameter. Substituting it out, we find that

$$\frac{(1-\chi)}{C_t^r} = \frac{(1-\chi)}{C_t^o} = \frac{\upsilon}{A_t L_t^r} = \frac{\upsilon}{A_t L_t^o} = \frac{\chi}{G_t},$$
(82)

which states that an efficient steady-state allocation implies that marginal utility of consumption across types of households and across alternative uses (public versus private goods) needs to be equal to the marginal utility of an additional unit of leisure across types. Using  $L_t = (1 - N_t)$ , we thus find that for an optimal steady state level of employment from a social planner's perspective, it holds that

$$\frac{\bar{Y}}{\bar{N}}\frac{(1-\chi)}{\bar{C}} = \frac{\upsilon}{(1-\bar{N})} \Rightarrow \frac{\bar{N}}{(1-\bar{N})} = \frac{1}{\gamma_C}\frac{(1-\chi)}{\upsilon},$$

which corresponds to equation (71) and, hence, is identical to the steady-state outcome in the competitive equilibrium with the labor subsidy at hand.

For the optimal distribution between public and private consumption goods, we make use of the fact that  $\gamma_G = 1 - \gamma_C$  resulting from  $\bar{Y} = \bar{C} + \bar{G}$ , the market clearing condition. Using equation (82), this can be transformed to  $\gamma_G = 1 - \frac{1}{Y} \left[ \lambda \bar{C}^r + (1 - \lambda) \bar{C}^o \right]$ , while we know from the first-order conditions of the social planner's problem that it must hold that  $\bar{C}^r = \bar{C}^o = \frac{(1-\chi)}{\chi} \bar{G}$ . Substituting in the previous equation, this implies  $\gamma_G = 1 - \frac{\bar{G}}{Y} \frac{(1-\chi)}{\chi} = 1 - \gamma_G \frac{(1-\chi)}{\chi}$ , which yields  $\chi = \gamma_G$ . Using this, the optimal labor supply just calculated and the optimal consumption level from the first-order conditions, we find that

$$\bar{N} = \frac{1}{1+\iota}$$

and

$$\bar{C} = \frac{(1 + v)}{(1 - \gamma_G)}$$

As shown above, this is equal to the solution obtained under the competitive equilibrium for  $\chi = \gamma_G$  (see equations (71) and (76)), which implies that the competitive equilibrium is thus an efficient steady state.

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