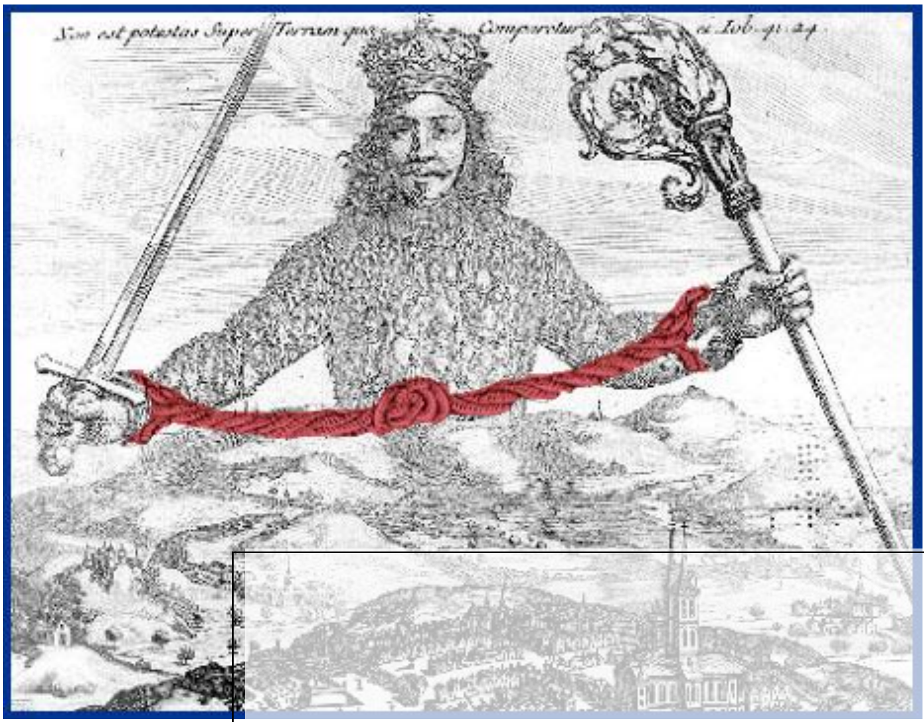

Constitutional Economics Network

Working Paper Series
ISSN No. 2193-7214

CEN Paper No. 01-2012

The rule of law and sustainability of the constitution: The case of tax evasion

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January 6, 2012

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The rule of law and sustainability of the constitution: The case of tax evasion*

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January 6, 2012

Abstract

Why do rulers play by the rules? We show that the legality requirement under the rule of law implements an endogenous enforcement mechanism supporting constitutionality. Agents which do not obey unconstitutional legal norms are not sanctioned under constitutional rule. A principal who defects from the constitution but cannot commit himself to never reinstall the constitution finds law enforcement more difficult. As more agents disobey, enforcement becomes less effective. The expectation of an eventual return to constitutionality becomes self-fulfilling. We show this mechanism to be effective in deterring a government from violating constitutional norms.

JEL codes: *D78, K10, K42, H26*

Keywords: *constitution, dynamic policy constraints, tax evasion, global games.*

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*For helpful comments we would like to thank Alberto Alesina, Moavia Algalith, Michael Chwe, Dinko Dimitrov, Cay Folkers, Ruvin Gekker, Ani Guerdjikova, Craig Mac-Caughrin, Francisco Moraiz, Wolfram Richter, Jonathan Thomas and participants at many seminars and workshops. Part of this paper was written while Gerald Pech was May Wong-Smith fellow at CRIEFF, University of St Andrews. Financial support is gratefully acknowledged.

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I. Introduction

No one is bound to obey an unconstitutional law and no courts are bound to enforce it (American Jurisprudence, Second Edition, Volume 16, Section 177).

Why does the government obey the constitution? The constitutional contract between government and citizens endows the government of the day with the means to force its will, whilst it does not provide for an exogenous enforcer equipped with an amount of "hard power" to match who ensures that the government plays by constitutional rules. At the same time, the constitution imposes constraints on the government, such as the requirement of "abiding by election results, rules governing policy choice, and a set of political rights of citizens" (Weingast, 1997) which from the government's perspective constitute a cost and should give it a temptation to free itself from constitutional constraints. The Pakistani Musharaf government's conflict with the supreme court in 2007 which resulted in the removal of non compliant judges, provides recent evidence of such temptation. The example also suggests that even governments whose position of power is not entirely rooted in the constitutional order have a preference for being seen as acting within an inherited constitutional framework.

Constitutional lawyers often stress the importance of the constitutional legitimacy of government action for the inner working of state institutions. Seen from this angle, a constitution establishing an order of the state (Maddox, 1982) constitutes an asset rather than a cost for the government. For constitutional legitimacy to represent an asset for the government, it must be true that citizens see it in their interest to contribute more to the objective of the government when the government acts in constitutionally legitimate rather than non legitimate ways. In that case, the government itself would have an interest of acting in legitimate ways and of abiding by constitutional rules. An example where such a mechanism has been intentionally designed to support lawful behavior by an agent of the government - the military - is an amendment of the code of conduct for US military personnel under the Carter administration which specifically states that soldiers have to execute "lawful" orders.[1]

The element of the constitutional order which formalizes the requirement of constitutional legitimacy is the rule of law, understood as the requirement that laws adhere legality and are compatible with the constitution (Hayek, 1960, p. 205). This constructive element is present in the constitution of practically every developed democracy with the possible exception of the British constitution.[2] In this paper we show that the incentives of the rule of law are such that the constitution is, indeed, an asset. The mechanism which drives this result is the negative consequence of the rule of law which says that unconstitutional laws are not enforced and citizens are not punished for not obeying such laws.[3] For this incentive mechanism to work, a future government must willingly choose to apply constitutional rules. If the future government is constitutional with positive probability, the rule of law gives incentives for the present government to abide by the constitution even with a finite planning horizon. This argument can be extended to infinite horizon economies where abidence by the rule of law becomes self-sustainable. To see how its incentives unfold, one has to imagine the rule of law

[1]We are greatful to Michael Chwe for providing this example.

[2]Because the (unwritten) British constitution establishes parliament rather than the people as sovereign, it cannot be seen as a contract between the people and the government in the usual sense. Therefore, the finding that the British constitution lacks the kind of enforcement mechanism which we study in this paper is entirely in line with the different role of its constitution.

[3]This contrasts with the positive consequence the rule of law where the present government ties the hands of the future government and increases its cost of abiding with the constitution (Pech, 2008).

as a device which attaches a label - constitutional - to a government which neither adopts unconstitutional laws nor enforces unconstitutional laws inherited by its predecessor. Say, the government adopts an unconstitutional law in period t and a citizen violates this law. In period $t+1$ the government may or may not sign up to the constitutional order. If it signs up, it must not enforce the law against the citizen. Only if it does not sign up it may act in an opportunistic way. The lack of constitutional legitimacy in the non constitutional state of the world results in a disadvantage in law enforcement whenever citizens attach a positive probability to the event that the constitutional order is reinstated. Ultimately, a government wants to reestablish the constitutional order, if all citizens expect that a successor government or the future agent of the present government wants to reestablish the constitutional order: If everyone expects to go unpunished, law enforcement will become prohibitively costly for the present government. But if the government tries to "buy" legitimacy by implementing the constitutional order, citizens' expectations become self-fulfilling.

That citizens' non compliant behavior has features of strategic complementarity and can ultimately result in the reversion of government policies has been demonstrated in the abandonment of the conscription law in Spain: As more and more young males refused to get drafted the government, due to overcrowded prisons, could not uphold a reasonable probability for offenders to get punished. Lohmann (1993) has demonstrated in a signaling game of collective action how the coordination of citizens' actions in open street protests could eventually result in the downfall of a government as experienced in the East German revolution. Whilst constitutional governments will often correct unconstitutional laws with the consequence of exempting previous trespassers from punishment, there are also examples of authoritarian regimes trying to buy legitimacy by subscribing to a democratic order. In the course of the transition from communism many governments, such as the Jaruszelsky regime in Poland, tried to negotiate a constitutional settlement with the opposition parties. In terms of the durability of laws and legal succession, issues of restitution of property collectivized during the communist reign have been on the agenda in all Eastern European transition countries. In particular, East Germany experienced the return to an original constitutional order with the German constitutional court (2001) ruling illegal all expropriations under the former East German regime. Finally, citizens' expectation of the eventual abandonment of an "unconstitutional" state of affairs adds to the pressure on the government. This has been demonstrated by the street protests of lawyers and the activities of opposition parties in Pakistan of 2007. In particular, the return to democratic rule held the promise of reinstating the chief justice of the constitutional court. One prediction of our theory is that a government would voluntarily sign up to the formal rules of a constitution which contains the rule of law as an element. Attempts by dictators to legitimize their rule using the cloak of the constitutional order even when doing so effectively constrains their choices can be understood from this perspective.[4]

Whilst the incentives from constitutional legitimacy and the lack thereof under the rule of law are straightforward, observed practice of implementing constitutional rule also creates incentives which pull in the opposite direction. Often, an incoming constitutional government not only protects those from punishment who have declined to comply with measures of a non democratic predecessor regime but also assuages those who were complying. Amnesties such as in the case of the Chilean transition in 1989 (see Barros, 2002) or the Spanish transition

[4] See Michalak/Pech (2008) on the constitutional choice problem of the Pinochet regime.

1977, whilst increasing support for the transition back to constitutional rule, also make compliant behavior under the non constitutional regime more attractive. In the limiting case, where compliance and non compliance are treated perfectly symmetrically, the choice of action of the citizen becomes independent of the probability of regime change. Such cases are rare in practice.[5] Rather, consider the case of a judge and her decision to challenge the government. The judge evaluates her seat and has an intrinsic pay off from challenging or not challenging the government. When she challenges the government she loses her seat under the non constitutional government but is reinstated under a constitutional government. If she does not challenge the government she keeps her seat. A judge who intrinsically prefers not to challenge the government over challenging the government will never challenge the government, irrespective of the probability of regime change. However, assume that the judge draws personal satisfaction from challenging the government, particularly, if it acts outside of the constitution. In that case, the prospect of losing her seat might deter the judge from challenging the government but the prospect of being reinstated after a regime change will make it more likely that she challenges. In order to be able to quantify the effect of legal rules on citizens with different preferences, we analyze the rule of law mechanism within an optimal taxation framework where the government faces a binding constitutional

[5] Consider the case of a conference participant who believes that he is unjustly charged a late fee of $350. At the normal rate of $250 he would be willing to present the paper. If the organizers change their policy, all are allowed to present at a price of $250. Will he decide to challenge the organizers when he assumes that his decision a negligible impact on their policy? Here, it is easy to see that the participant pays the late fee whenever he prefers presenting at a price of $350 over not presenting at a price of 0, independent of the probability of policy change.

constraint on its income tax rate[6] and citizens truthfully report their income or evade taxes. The application of our theory to the problem of enforcing constitutionality in taxation is particularly appealing. As the British poll tax revolt has shown, disobedient behavior by tax payers can have no less dramatic effects than the previously listed examples.

We model our agents as long-lived and forward looking, and the government as opportunistic and attempting to maximize its long term net revenue increased by what we call a defector's rent. The defector's rent is earned each period the government acts outside of the constitution. It may be positive (like income from appropriated national resources) or negative (like international sanctions or an intrinsic cost). Citizens and government face uncertainty when making their decisions. The government only imperfectly forecasts the defector's rent before it decides on whether or not to breach the constitution. Once it switches back, it not only has to accept the constitutional tax rate applied to a diminished tax base. It also has to sacrifice income from fining tax evaders. As a consequence, it does worse than if it had stayed constitutional in the first place. Therefore, defecting from the constitution is more risky under the rule of law than without it. Citizens have only received a noisy signal of the actual realization of the defector's rent when they submit their income reports. This kind of uncertainty is the reason why citizens hold non degenerated probabilities over the behavior of their fellow citizens and allows us to rule out equilibria which rely on unrealistically strong assumptions about citizens' ability to fully co-ordinate behavior. We uniquely obtain a critical realization of the defector's rent below which the government returns to the constitution and we can show that the rule of law reduces incentives to violate the constitution

[6] In California, proposition 13 puts a 1% cap on the property tax rate. The German constitutional court has recently interpreted the constitution as restricting the power to tax to 50% of any income.

in the first place.

In order to focus entirely on the incentive mechanism under the rule of law we ignore all other details of constitutional practice. We do not consider alternative enforcement mechanisms such as elections. Furthermore, we consider the fact of a transgression to be self-evident. Thus there is no role for the constitutional court in establishing that the government is in violation of the constitution and there is no distinction between formal and material adherence to constitutional rules. Modelling the government as a surplus maximizer appears to us as the most natural way of formulating its objective function, even though in practice there might be the need to share the surplus with other groups in society. A probability of staying in power of less than one can be accommodated by the government's discount factor. An interest in the government's long term revenue provides the most direct link between the future government's actions and the realization of today's government's objectives. As Pech (2008) demonstrates, the enforcement of property rights by a future government can indirectly provide such as link.

An earlier approach which shows that a social contract has properties for which it qualifies as an asset is by Kotlikoff/Persson/Svennson (1988). They show that a social contract forbidding expropriative taxation is adhered to if supported by transfers from the young generation to the old. However, their construction does not rest on a particular feature of constitutional law. Our own sustainability argument also has features in common with other approaches to the time-inconsistency problem such as Chari/Kehoe (1990) who show that the government not necessarily reneges on the tax rate it has announced. Whilst those contributions show that adherence to some standard can be supported in equilibrium, they rely on trust or agreement which is costly to achieve. In our approach the public faces a different co-ordination problem and our commitment technology uses a dynamic legal constraint to support the outcome. Our formal model draws on the results on global games (see Morris/Shin, 2000).

More recently, a number of contributions have related political outcomes to the constitutional choice problem from a normative and positive angle (see, for example, Aghion/Alesina/Trebi, 2004, Voigt, 1997). Gersbach (2005) has studied the design of ideal constitutional rules for voting and agenda setting and the role of incentive contracts for rule adherence by elected politicians. Our paper adds to a rapidly growing literature which shows how favorable equilibria of the political process can be supported. Weingast (1997, 2005) derives self-enforcing equilibria in which social groups are able to coordinate against government transgressions. He refers to constitutional standards as red lines for co-ordinating citizens' actions against violations by the sovereign. Neumärker (2005) gives conditions under which an autocratic government is fended off by coordinating citizens. Dixit, Grossman and Gul (2000) derive properties of a Markov perfect sharing equilibrium in two-party competition with an exogenous process by which political power is assigned. Lagunoff (2001) establishes that tolerant legal standards can be supported in a legal-political game with errors in law enforcement. Gersbach (2004) shows that the one-person one-vote rule is unanimously acceptable when it is compared to a rule where disenfranchisement may be put on the political agenda.

Our paper is organized as follows: Section II gives an overview of the structure of the game. In section III we set up the decision problems of the citizens and the government. In section IV we consider the decision of a violating government to switch back to the constitution under common knowledge and under imperfect information. Section V derives the critical value of the defector's rent in the decision to defect from the constitution and extends our results to an infinite horizon game. Section VI concludes.

II. Outline of the Game

Figure 1 depicts the game between the public and a government which has been constitutional in period $t-1$. At the beginning of period t, the tax policy is selected, income is earned and reported. All income is spent, taxed and tax evaders detected at the end of the period. With the selection of its tax rate, the government implicitly selects its constitutional state: A defecting governments selects a tax rate τ above the constitutionally permissible rate, τ^c. The defector's rent k_t is the current lump sum pay off for a defecting government. It follows a process with $Ek_t = k_{t-1}$. In period t, the defector's rent for $t-1$, k_{t-1}, is common knowledge. Together with the government's current constitutional state $K \in \{nc, c\}$, k_{t-1} represents the entire history of the game.

In the beginning of period t, information is revealed as follows: after selecting τ, the government learns k_t,[7] each citizen i receives a noisy signal x^i of k_t and subsequently reports her income. Based on aggregate income data the government computes tax evasion. At the end of period t a defector government considers its constitutional state. If it does not reform, the government consumes its defector's rent. By reforming, on the other hand, a government can escape a negative rent. At this point, the government also selects the detection probability r which tax evaders face and a fine on detected, undeclared income.

[7]The assumption that when setting the tax rate, the government does not have an information advantage rules out incentives for signalling which could result in implausible equilibria, see Cho and Kreps (1987).

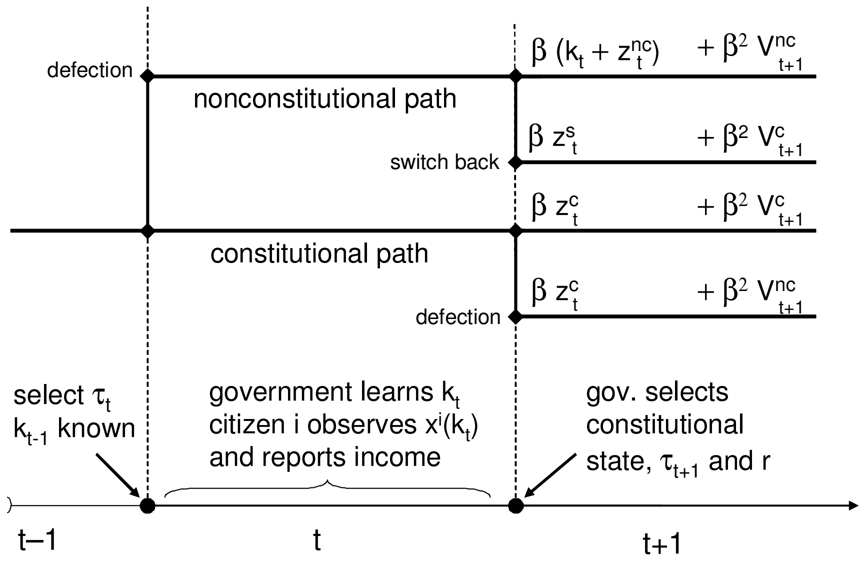

Figure 1: Game between government and citizens

Revenue in t net of the defector's rent is z_t^{nc} for the non constitutional government, z_t^c for the constitutional government and z_t^s for a government which switches back to the constitution. z_t^{nc}, z_t^c and z_t^s are finite. The government discounts its income at a rate $\beta \in (0,1)$. Expected continuation pay off on the constitutional path - evaluated at the beginning of $t+1$ - is βV_{t+1}^c. Continuation pay off on the non constitutional path is βV_{t+1}^{nc}. The rule of law binds as follows: Only by levying a fine not exceeding the evaded constitutional tax and by announcing a tax rate not exceeding τ^c for $t+1$ does a reforming government regain constitutional status.[8]

With the exception of switching back according to constitutional rules, governments are

[8]If the reformed government would newly assess all citizens for tax purposes it would treat previously complying and non complying citizens symmetrically and the decision to evade taxes under the non constitutional regime would not depend on the probability of regime change. If there is a factual or legal advantage to those who did underdeclare, the decision to evade does depend on the probability of regime change.

able to commit to their pre-announced tax policies for the current period.[9] In the other case we would encounter in our model the same problem of time-inconsistent taxation as is well known in the context of capital taxation. With a limited time horizon, each government would want to impose a tax rate of 1 ex post and the argument for being constitutional would unravel. With an infinite planning horizon, on the other hand, a tax rate below 1 can be shown to be sustainable if trust matters (see Chari/Kehoe, 1990).

III. The Basic Model of Tax Evasion

In this section we set out the basic model of tax evasion. This is a simplified multi-agent version of Allingham/Sandmo (1972) and Kolm (1973). For the current period t we drop all time indices. A citizen from the income earning population earns an income of 1, other citizens earn an income of zero. The true size of both populations is known to the government. Citizens from the income earning population either fully declare their income or declare an income of zero.[10]

[9] Otherwise it is not restrictive that the defector can reconsider its constitutional state whilst the non defecting government cannot. We explicitly allow that the latter can stage a defection based on $t+1$-information in the following period. Furthermore, after any defection must there necessarily come one point in time when the defector can reconsider its state using up-dated information.

[10] Such strategies are supported in the Bayes- Nash equilibrium of an income reporting game were all citizens report 0 or 1 and the government believes with a probability of 1 that a citizen reporting y$\in (0,1)$ is a tax evader with probability 1.

A. A Citizen's Problem

This section analyzes the optimal income reporting strategy for an income earner. Income earning citizens are risk neutral. They maximize expected net income increased by a honesty measure η^i. A citizen who honestly reports her income receives a psychic pay off of η^i. Under a government which is not a reforming government, the expected payment of a tax evader is equal to the detection probability which citizen i expects of government $K \in \{c, nc\}$, $\widehat{r}^{Ki}$, and the expected payment of a tax payer is simply τ^K. Under a reforming government s, the expected payment of a tax evader is $\tau^c \widehat{r}^{si}$ and expected payment of a tax payer is τ^c.

Suppose the government has defected from the constitution and let P^i be the citizen's belief that the government will reform itself. If the citizen declares her income, her expected payment is $(1-P^i)\tau^{nc}+P^i\tau^c$ whilst her expected payment if she evades is $(1-P^i)\widehat{r}^{nci}+P^i\tau^c\widehat{r}^{si}$. Breaking indifference in favor of evasion, a citizen evades taxes if

$$\Phi^i \equiv (1-P^i)[\tau^{nc} - \widehat{r}^{nci}] + P^i(1-\widehat{r}^{si})\tau^c - \eta^i \geq 0. \tag{1}$$

Of the distribution of the parameter η^i in the income earning population we assume that it is equally distributed on $[\underline{\eta}, \overline{\eta}]$. Heterogeneous preferences guarantee a non degenerate optimal tax policy even in the case where citizens hold homogenous beliefs.

By (1), the share of tax evaders in terms of the whole income earning population is

$$\theta(\widetilde{P}, \tau^{nc}, \widetilde{r}^{nc}, \widetilde{r}^{s}, \tau^c | k_t) = \int_{i|\Phi^i \geq 0} di \tag{2}$$

where $\widetilde{P}$ is the distribution of beliefs P^i of the citizens and $\widetilde{r}^{nc}$ and $\widetilde{r}^{s}$ are distributions of estimated detection probabilities in accordance with a value of k_t. From (1) it is immediate that $\frac{\partial \theta}{\partial \tau} > 0$. A mild assumption which we discuss in section C. below ensures that θ increases as $\widetilde{P}$ shifts to some $\widetilde{P}'$ which first order stochastically dominates $\widetilde{P}$.

B. The Government's Problem

The goverment's instruments fine σ, tax rate τ and and detection probability r satisfying $(\sigma, \tau, r) \in [0,1]^3$. Immediately before the beginning of period $t+1$, the government selects r and σ for a given choice of tax rate τ' in period t and a given aggregate level of tax evasion, θ', in order to maximize detection receipts over detection costs $C(r, \theta)$:

$$\max_{\sigma, r} z = \theta' r\sigma + (1-\theta')\tau' - C(r, \theta') \text{ s.t. } \sigma \leq \sigma^{\max}, \tag{3}$$

where $\sigma^{\max}$ is the greatest legally admissible and feasible fine. We assume that detection cost C is strictly convex in both arguments and $C(0,.) = 0$. Partial derivatives are denominated C_θ and C_r, second derivatives C_{rr}, $C_{r\theta}$ and $C_{\theta\theta}$. Moreover:

Assumption 1 *The cost function satisfies* $C_{r\theta} > 1$ *and* $C(r, 1) > r$ for $r \in (0, 1]$.

The first part of assumption 1 states that the marginal cost of securing a given detection probability increases by more than 1 if the share of evaders increases. The consequence of this assumption is that as more citizens evade taxes an optimally adjusting tax administration will allow the individual detection probability to decrease. This is especially plausible in the short run where the tax authority is likely to act under a capacity constraint. The second part of assumption 1 states that if everybody evades taxes, detection becomes prohibitively costly.[11]

[11] For our results it would be sufficient that it is prohibitively costly to raise revenue from detection beyond some critical level which is in accordance with the argument that the tax authority's resources cannot be readily expanded in the short run.

Because the government cares about revenue and because the fine is selected after the citizens have submitted their income reports, no government can benefit from selecting a smaller than the maximally permissible fine. Therefore, every government wants to charge the maximally feasible and admissible fine, $\sigma^{\max}$. For a non reforming government that fine is $\sigma = 1$. This gives the optimal detection effort r^K for $K \in \{c, nc\}$ as

$$\begin{aligned} C_r(r^K, \theta) &= \theta \Rightarrow r^K = \min\left\{C_r^{-1}(r^K, \theta), 1\right\} \text{ for } 0 < \theta \leq \overline{\theta}, \\ r^K &= 0 \text{ if } \theta > \overline{\theta} \text{ or } \theta = 0, \end{aligned} \tag{4}$$

where $\overline{\theta}$ is the level of evasion above which the government gives up its detection efforts as θ increases. Because $C(r, 1) > r$ for $r > 0$ and $C_{\theta\theta} > 0$ such $\overline{\theta}$ uniquely exists.

By the rule of law constraint, a reforming government sets the fine $\sigma = \tau^c$. Its optimal detection policy is implicitly given by $C_r(r^s, \theta) = \tau^c\theta$ for $\theta \leq \overline{\theta}'$, the maximum share of evaders compatible with a positive net revenue from detection, $r^s\tau^c\theta \geq C$, and $r^s = 0$ else. It is immediate that $\overline{\theta}' < \overline{\theta}$ and $r^s < r^{nc}$ for $r^{nc} \neq 0$. (2), (4) and the policy rule for r^s define a taxation-detection equilibrium $\left\langle \theta(\tau^{nc}, \tau^c, \widetilde{r}^{nc}, \widetilde{r}^s, \widetilde{P}|k_t), r^{nc}(\theta(\cdot)), r^s(\theta(\cdot)) \right\rangle$. In this equilibrium, distributions of beliefs, $\widetilde{r}^{nc}$ and $\widetilde{r}^s$, result from the best estimates $\widehat{r}^{nci}$ and $\widehat{r}^{si}$ of individual citizens given their information set and the government acting in accordance with its appropriate policy rule r^{nc} or r^s.

Regarding the tax selection problem, we claim that an optimal tax policy with $\tau^{nc} > \tau^c$ exists if the government is sufficiently determined to violate the constitution (see appendix II). Because citizens have heterogenous preferences, the optimal tax problem is well-behaved even if the noise in citizens' observations vanishes.

C. Regularity conditions

The effect of P^i on the decision to evade is ambiguous. This we find by differentiating the condition for evading, (1), with respect to P^i. It is apparent that P^i has a positive effect if the cost of declaring taxes in the non-constitutional state is smaller than the cost of declaring taxes in the case of a switch back which we require:

Assumption 2 *The optimal tax policy and the detection policies in the two states satisfy*

$\tau^{nc} - \widehat{r}^{nci} < (1 - \widehat{r}^{si})\tau^c$ *for every citizen* i.

The economic reason why the effect of P^i on tax evasion is ambiguous is that in the case of a switch back not only is the fine abandoned - which encourages tax evasion - but excessive taxes are returned to the citizen which encourages honest reporting. As we are going to show in proposition 3, in equilibrium the government wants to select a policy which indeed fulfills assumption 2 if the noise in the citizens' observations is small. So our assumption 2 is consistent with equilibrium behavior.

The following assumption on the optimal detection rate in the case of a switch back ensures that tax evasion feeds back on government policy:

Assumption 3 *The optimal tax policy and detection policy after a switch back satisfy*

(a) $r^s < \frac{\tau^{nc} - \tau^c}{(1-\tau^c)}$ *and (b)* $r^s < 1 - \frac{\overline{\eta}}{\tau^c}$.

The first condition ensures that tax evasion marginally increases the switch back probability in the proof of lemma 2. Whilst appearing strong and not a condition on the primers of the model, it is clear that there always is a value for marginal cost C_r such that $r^s = 0$. Condition (a) is more general because some $r^s > 0$ is admissible which can be obtained by imposing a corresponding condition on C_r. The second condition places another upper boundary on r^s. This one is easily fulfilled if $\overline{\eta}$ is sufficiently close to zero. Effectively, part (b) of assumption 3 rules out the possibility that citizens enjoy paying taxes to such an extent that they are honest even if they know that the government switches back with certainty and ensures that $\theta(P = 1) = 1$.

IV. Switching back to the constitution

Say the government has defected from the constitution in t. By the end of t, is has learned the level of aggregate tax evasion and its defector's rent. If the government does not reform, it earns a current income of $k_t + z_t^{nc}(\theta_t)$ and a continuation income of $V_{t+1}^{nc}(k_t)$. If it reforms, it earns in the current period a transition income $z_t^s(\theta_t)$ and a continuation income of V_{t+1}^c. The government switches back if the expected pay off difference along the constitutional path over the non-constitutional path, δ_t, is positive:

$$\delta_t = z_t^s(\theta_t) + \beta V_{t+1}^c(k_t) - z_t^{nc}(\theta_t) - \beta V_{t+1}^{nc}(k_t) - k_t > 0. \tag{5}$$

There is always some $\underline{k}$ such that (5) is positive and the government switches back even if the realization of tax evasion is at its lower boundary $\underline{\theta}$. On the other hand, there is $\overline{k}$ such that the government does not even switch back if everybody evades taxes, $\theta = 1$. In the intermediate range $(\underline{k}, \overline{k})$, the switch back decision depends on tax evasion. It remains to show that $\underline{k} < \overline{k}$. We derive this and some of the following results for the truncated version of the game with two periods. In the proof of proposition 5 in the appendix we extend all results to an infinite horizon. In the truncated version of the game $T = t + 1$ is the last period where the government carries out its announced policy. The continuation pay off is

$V_T^c = z_T^c$ along the constitutional path and $V_T^{nc} = E\left(k_T + z_T^{nc}\right)$ along the non constitutional path where $Ek_T = k_{T-1}$.

Lemma 1 *In the truncated game, there is* $\underline{k}$ *such that a government wants to switch back even if* $\theta = \underline{\theta}$ *and there is* $\overline{k} > \underline{k}$*, such that a government does not even want to switch back if* $\theta = 1$.

Proof. See part A of the appendix ■

A. Multiple equilibria under common knowledge

Under common knowledge the parameter k_t can be perfectly observed by the citizens. We construct a Nash equilibrium in the following way: Given the strategies of the other citizens and the government, no citizen wishes to change her strategy. Furthermore, given the strategies of the citizens, the government wishes to carry out its policy. Focusing on equilibria in pure strategies we obtain:[12]

Proposition 1 *Under common knowledge, the following combinations of beliefs and strategies constitute an equilibrium in pure strategies: For* $k \leq \underline{k} : \langle P = 1, \text{ switch back}\rangle$*. For* $k \in (\underline{k}, \overline{k}) : \langle P = 1, \text{ switch back}\rangle$ *and* $\langle P = 0, \text{ not switch back}\rangle$*. For* $k \geq \overline{k} : \langle P = 0, \text{ not switch back}\rangle$*.*

This result follows immediately from the definition of equilibrium and lemma 1, noting that $\theta(P = 1) = 1$ and $\theta(P = 0) = \underline{\theta}$. For $k \geq \overline{k}$ the government switches back even if all citizens evade and, consequently, all citizens do want to evade. For $k \leq \underline{k}$ the government does switch back even with tax evasion at its lower boundary $\underline{\theta}$ and so only a share $\underline{\theta}$ of

[12]There is another, unstable equilibrium in which the government plays a mixed strategy, see a similar result in Verdier/Roland (2003).

citizens actually want to evade. If k_t is in the intermediate range $(\underline{k},\overline{k})$, the government's equilibrium strategy depends on P and the game has multiple equilibria.

B. Unique equilibrium under incomplete information

The assumption of common knowledge is in itself very strong and the phenomenon of multiple equilibria which it implies prevents us from proceeding at that point because we cannot in a systematic way assign probabilities to the events - switch back or not switch back - which these equilibria imply. We know, however, that in the end one event will take place and that agents have to find ways to evaluate the risk involved in some way ex ante. As the theory of global games has shown, relaxing the assumption of common knowledge removes the problem of multiple equilibria and allows us to treat the formation of expectations over possible events in a systematic way.[13] Under incomplete information citizens cannot perfectly observe k_t when they decide over tax evasion. Instead each citizen observes a distinct signal x^i which is uniformly distributed on $(k_t - \varepsilon, k_t + \varepsilon)$. Citizens have a dominant strategy when they know that $k_t \leq \underline{k}$ or $k_t \geq \overline{k}$, which is true if they receive a signal which is at most $\underline{k} - \varepsilon$ or higher than $\overline{k} + \varepsilon$. In order to derive equilibrium strategies in the intermediate range we have to establish first that citizens' decisions over tax evasion are strategic complements throughout:

Lemma 2 *If assumption 3 (a) is fulfilled, tax avoidance strategies are strategic complements.*

Proof. See part B of the appendix ■

[13] See Morris/Shin (2000) for an overview. The solution of a global game coincides with the risk dominant solution and can be justified because of that.

Assumption 3 (a) ensures that the withdrawal effect of tax evasion on the defector's income, z^{nc}, is greater than the withdrawal effect on switch back income, z^s.[14] With strategic complementarity, tax evasion precipitates a switch back in (5). On the other hand, the critical mass of evaders necessary to fulfill (5) increases in k_t:

Lemma 3 *There is a critical mass of evaders $\phi(k_t)$ for which $\delta_t = 0$ and which is strictly increasing in k_t with $\phi(\overline{k}) = 1$ and $\phi(\underline{k}) = \underline{\theta}$.*

Proof. See part C of the appendix ■

Because tax avoidance strategies are complements in the unstable region of k_t we can iteratively eliminate dominated strategies starting at the upper and lower boundaries of the dominance regions. A citizen's strategy takes the form: evade taxes if the signal x^i is smaller than a threshold ξ^i which in turn depends on her preference parameter η^i. We can show:[15]

Proposition 2 *In the incomplete information truncated game there is a unique equilibrium point k^* supported by a distribution of individual thresholds ξ^i, $\widetilde{\xi}$, such that $\underline{k} < k^* < \overline{k}$ and the government switches back if $k_t < k^*$.*

Proof. See part D of the appendix ■

In part E of the appendix, we obtain comparative statics results for the limiting case $\varepsilon \to 0$, $|\underline{\eta}, \overline{\eta}| \to 0$. The closer k^* is to $\underline{k}$, the greater is the probability which the marginal tax evader assigns to a switch back. This probability has to be greater with a more convex cost-function in θ, a lower τ^c, a greater tax honesty η and a lower tax-detection differential under

[14] A high τ^c, which reduces the incentive to violate the constitution, could weaken the switch back mechanism through a withdrawal effect in conjunction with a high r^s.

[15] Frankel/Morris/Pauzner (2003) derive a uniqueness result in a setting with finitely many types and continuous actions for vanishing noise.

the constitution. Intuitively, if the government's cost function does not increase sharply for low levels of evasion, citizens need more assurance that the government wants to switch back. The same is true if η is relatively high. If, in turn, the probability of a switch back is high, tax evasion behavior will be mainly determined by the differential charge on declared income in the case of a switch back, $(1 - r^s)\tau^c$. If, on the other hand, the differential charge on declared income in the non constitutional case $(\tau^{nc} - r^{nc})$ is high, citizens are willing to evade taxes even if the probability of a switch back is rather low. It is worth noting that the overall effect of τ^c on the threshold value k^* is ambiguous, as a higher value of τ^c results in a higher value of $\underline{k}$ from the government's switch-back condition (5).

Finally, we can show that in the case where the noise in the citizens' observation vanishes, the government wants to set the tax rate such that assumption 2 is fulfilled for all citizens:

Proposition 3 *For $\varepsilon \to 0$, the government chooses τ such that for all citizens the condition $\tau^{nc} - \widehat{r}^{nci} < (1 - \widehat{r}^{si})\tau^c$ is always fulfilled ex post.*

Proof. See part F of the appendix ■

The intuition behind proposition 3 is that the government wants to make sure that at least the most willing citizen pays taxes. If it cannot make sure that this citizen pays taxes, nobody will and the setting of τ is irrelevant. In the more general case, $\varepsilon > 0$, the government might face a trade-off between taxing the citizen with the lowest signal and the most willing citizen. Therefore, we make this claim only for vanishing noise. The proposition holds without qualification if the government does not undertake any detection effort after the switch back.

V. The Decision to Defect from the Constitution

Having established conditions under which a defected government wants to reform itself we now analyze the decision to defect from the constitution in the first place. We determine the critical value of k_{t-1} for a defecting government in the case where the noise in the citizens' observation, ε, vanishes. As a benchmark, we first determine the critical k_{t-1} at which the government deviates from the constitution in the absence of the rule of law. Here, the government can freely choose its constitutional status and it always enforces its policy. The government stays constitutional if

$$\beta \left[z_t^c - z_t^{nc} - k_{t-1} \right] \geq 0 \tag{6}$$

Let k^0 be the value for which (6) is binding. $k^0 < 0$ because, by assumption, the constitutional tax rate is binding.

Now assume that the rule of law is in operation and the government in $t-1$ knows that it might want to switch back depending on the realization of k_t. Let k_t be a random variable which is uniformly distributed on $(k_{t-1} - \Delta, k_{t-1} + \Delta)$. This process is common knowledge. If the error term in the signal vanishes, citizens' prior knowledge of k_{t-1} does not affect their expectations after receiving a signal of k_t:

Lemma 4 *For $\varepsilon \to 0$, the equilibrium point in the game with a prior k_{t-1}, $\widehat{k}_t^*$ and the equilibrium point in the game without a prior, k^*, coincide.*

Proof. See part G of the appendix ■

The continuation pay off along the constitutional path is βV_t^c and the continuation pay off on the non-constitutional path is βV_t^{nc}. The government stays constitutional if at k_{t-1}

$$\beta\left[V_t^c(k_{t-1}) - V_t^{nc}(k_{t-1})\right] \geq 0. \tag{7}$$

Denominate k^{**} the critical value for which (7) is binding. Using k^* and the density function of k_t for given prior k_{t-1}, $\omega(k_t|k_{t-1})$, we can express expected pay off along the non-constitutional path recursively as

$$V_t^{nc}(k_{t-1}) = \int_{k_t<k^*} \omega(k_t|k_{t-1})\left[z_t^s + \beta V_{t+1}^c(k_t)\right] dk_t + \int_{k_t\geq k^*} \omega(k_t|k_{t-1})\left[z_t^{nc} + k_t + \beta V_{t+1}^{nc}(k_t)\right] dk_t. \tag{8}$$

The first term on the right hand side is the contribution of income earned in the case of a switch back and the second term is the contribution of income earned if the government stays on the non-constitutional path. We can use the fact that ε vanishes so all citizens evade taxes for $k_t < k^*$ with resulting pay offs $z_t^s = 0$ and $z_t^{nc}(1, k_t) = 0$ whilst $\underline{\theta}$ evade for $k_t \geq k^*$ giving $z_t^{nc}(\underline{\theta}, k_t)$. As a consequence, the right hand side of expression (7) is continuous in k_{t-1}. In order to evaluate the pay off along the constitutional path we need to know the decision criterion employed by future agents of the government in their decision over a defection from the constitution. For now we assume that this decision criterion is given by the rule: defect in period s if $k_{s-1} > k^{**\prime}$ for $s > t$ and keep with the constitution otherwise. We get

$$V_t^c(k_{t-1}) = z_t^c + \beta\left[\int_{k_t\leq k^{**\prime}} \omega(k_t|k_{t-1}) V_{t+1}^c(k_t) dk_t + \int_{k_t>k^{**\prime}} \omega(k_t|k_{t-1}) V_{t+1}^{nc}(k_t) dk_t\right] \tag{9}$$

where z_t^c is the constitutional pay off on the constitutional path in t, the first integral gives the continuation pay off under the constitution weighted with the probability of staying constitutional in $t+1$ and the second integral giving the contribution of income realized after defecting in $t+1$. Because $\beta > 0$, the critical value k^{**} is governed by the difference $D_t(k_{t-1}) = V_t^c(k_{t-1}) - V_t^{nc}(k_{t-1})$ in condition (7). Using the fact that $z_t^s = 0$ and $\omega(k_t|k_{t-1}) = \omega$ for $k_t \in [k_{t-1} - \Delta, k_{t-1} + \Delta]$, we can write this difference for $k^{**\prime} \geq k^*$ recursively as

$$D_t(k_{t-1}) = z_t^c - \int_{\max(k^*, k_{t-1}-\Delta)}^{k_{t-1}+\Delta} \omega[z_t^{nc} + k_t]dk_t + \beta \int_{\max(k^*, k_{t-1}-\Delta)}^{\min(k^{**\prime}, k_{t-1}+\Delta)} \omega D_{t+1}(k_t)dk_t. \tag{10}$$

The last term on the right hand side can be interpreted as a lock-in effect into the non constitutional state: Suppose that after a defection in the beginning of t the government realizes $k_t \in [k^*, k^{**\prime})$. It now regrets having defected in t when in $t+1$ it could have earned a higher pay off under the constitution. After a defection in t, however, it does not want to perform a switch back which is governed by k^*. In the range $(k^*, k^{**\prime})$, $D_{t+1}(k_t)$ is positive. So the lock-in-effect works as an additional deterrent against a defection.

On the other hand, if stakes are sufficiently high, the government may face a potentially volatile situation where $k^{**\prime} < k^*$.[16] In that case we replace the last term on the left-hand side of (10) by $-\beta \int_{\max(k^{**\prime}, k_{t-1}-\Delta)}^{\min(k^*, k_{t-1}+\Delta)} \omega D_{t+1}(k_t)dk_t$. If $k_t \in (k^{**\prime}, k^*)$ the government switches back and starts in the constitutional situation in period $t+1$ when it could have realized a higher expected pay off by defecting in $t+1$, had it shown restraint in t. $D_{t+1}(k_t)$ is negative in the range $(k^{**\prime}, k^*)$, so again the lock-in-effect deters the government from defecting from the constitution.

[16]See step 4 in the proof of proposition 5.

A. The truncated game

Consider the truncated game with a last period $T = t + 1$. In T, every government defects if (6) is violated. Therefore, we have $k^{**\prime} = k^0$, $V_T^{nc} = z_T^{nc} + k_T$ and $V_T^c = z_T^c$. Now it is straightforward to show when the rule of law economically matters. Comparing (6) and (7) for period $T - 1$, we find that the latter condition results in a higher cut off point whenever the government's uncertainty about its future preferences (which is why it might have to switch back) is overriding its concerns about its constitutional pay off:

Proposition 4 *In the truncated game, for* $\frac{1}{1+\beta} z^c < \Delta$ *the switch back mechanism matters: The critical value above which the government defects in the absence of the rule of law,* k^0*, is smaller than the critical value under the rule of law,* k^{**}*.*

Proof. See part H of the appendix. ■

It is intuitive that the condition of proposition 4 is fulfilled for sufficiently large values of Δ: the government needs to face some risk that it will have to perform a switch back. In the truncated game, $\frac{1}{1+\beta} z^c$ equals the distance $k^0 - k^*$: The current pay off z^c enters the decision to defect but for the decision to switch back it is a "bygone". The rule of law matters, if the switch back point k^* is less than Δ away from k^0, which implies that at $k_{T-2} = k^0$ a subsequent switch back cannot be ruled out.

B. The infinite horizon game

We construct an equilibrium for the infinite horizon case in the following way: Assume that all future governments follow a defection rule $k^{**\prime}$. Then determine a switch back point k^* and a defection value k^{**} for the current government. A stationary value $k^{**} = k^{**\prime}$ is a focal point of the infinite horizon game where each government selects k^{**} assuming that subsequent governments will select k^{**} as well.[17]

Proposition 5 *In the infinite horizon game there uniquely exists a stationary value k^{**} above which each government violates the constitution.*

Proof. See part I of the appendix. ■

VI. Discussion

In this paper we have analyzed one particular aspect of the rule of law - the dynamic policy constraint which results from the legality requirement - and have shown that it supports co-ordinated actions one the side of citizens and deters the government from violating the constitutional order. The rule of law can thus be seen as an element of an endogenous enforcement mechanism. It has an economic effect because the government realizes that its future agent may contribute to its own punishment.

Our analysis has been facilitated by the assumption of vanishing noise in the citizens observations. As a consequence, almost certainly everybody evades taxes for $k_t \leq k^*$ and the government does not have to return any excess revenue. If there are pay back obligations which accumulate over time, a non constitutional government would eventually lock itself out of the constitution. Whilst with a farsighted government such a lock-out effect works as a deterrent against defecting, citizens might be willing to renegotiate on the constitution, once a defection has occurred. This raises issues of constitutional reform in dictatorship (Michalak/Pech, 2008) which are beyond the scope of the present paper.

[17] k^{**} corresponds to an equilibrium where the government plays a Markov perfect strategy.

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VII. Appendix I

In part A-D of the appendix we set up the post-defection game under asymmetric information. We extend the standard approach (Morris/Shin, 2000) in order to deal with heterogeneity of preferences which ensures a non-degenerate optimal taxation problem.

A. Range of k: Proof of lemma 1

In order to derive dominance regions for the citizens we need to determine the range of k for which the decision of the government does not depend on tax evasion. Let $D_{t+1}(k) := \beta\left(V^c_{t+1}(k) - V^{nc}_{t+1}(k)\right)$. In the truncated game, $D_T(k) = \beta E_{|k_{T-1}=k}(z^c_T - z^{nc}_T - k_T)$. In the last period, the government does not switch back, so z^{nc}_T does not depend on k_T and we have $D_T(k'') - D_T(k') = \beta(k' - k'')$. Define $\underline{k} = D_T(\underline{k}) + z^s(\underline{\theta}, \tau^c, \sigma^s) - z^{nc}(\underline{\theta}, \tau^{nc}, \sigma^{nc})$ with the switch-back revenue $z^s(\underline{\theta}, \tau^c, \sigma^s)$ and $\overline{k} = D_T(\overline{k}) - z^{nc}(1, \tau^{nc}, \sigma^{nc})$. Therefore, $\overline{k} - \underline{k} = D_T(\overline{k}) - D_T(\underline{k}) - B$ with $B = z^s(\underline{\theta}, \tau^c, \sigma^s) - z^{nc}(1, \tau^{nc}, \sigma^{nc}) + z^{nc}(\underline{\theta}, \tau^{nc}, \sigma^{nc})$. We can show that $B < 0$: Because, $z^{nc}(1, \tau^{nc}, \sigma^{nc}) = 0$ it suffices to show that $z^s(\underline{\theta}, \tau^c, \sigma^s) < z^{nc}(\underline{\theta}, \tau^{nc}, \sigma^{nc})$.

Using $z_2^s = \sigma^s\underline{\theta}r^s - C(r^s, \underline{\theta})$ and $z_2^{nc} = r^{nc}\sigma^{nc}\underline{\theta} - C(r^{nc}, \underline{\theta})$ we can write $z^s(\underline{\theta}, \tau^c, \sigma^s) = (1-\underline{\theta})\tau^c + \max[0, z_2^s]$ and $z^{nc}(\underline{\theta}, \tau^{nc}) = \tau^{nc}(1-\underline{\theta}) + \max[0, z_2^{nc}]$. The claim holds because: (a) $\tau^{nc} > \tau^c$ for a defecting government and (b) $z_2^{nc}(r^{nc}, \underline{\theta}, \sigma^{nc}) \geq z_2^s(\underline{\theta}, r^s, \sigma^s)$. To see that (b) holds note that the nc-government chooses (σ^{nc}, r^{nc}) to maximize z_2 under the constraint $\sigma \leq 1$ whilst for the s-government $\sigma \leq \tau^c < 1$. Because (σ^s, r^s) is feasible for the nc-government the claim follows. With $B < 0$ we have $\overline{k} - \underline{k} > D_T(\overline{k}) - D_T(\underline{k})$. Using $D_T(\overline{k}) - D_T(\underline{k}) = \beta(\underline{k} - \overline{k})$ gives $(1+\beta)(\overline{k} - \underline{k}) > 0$ and, therefore, $\overline{k} > \underline{k}$ for all positive β.

B. Strategic complementarity: Proof of lemma 2

From (1), tax avoidance strategies are strategic complements if $\frac{\partial \Phi^i}{\partial P^i} > 0$, which is ensured by assumption 1, and the policy rules for the government satisfy $\frac{\partial \widehat{r}^{nci}}{\partial \theta} < 0$, $\frac{\partial \widehat{r}^{si}}{\partial \theta} < 0$ and $\partial P^i / \partial \theta > 0$. The condition on the detection probabilities follows directly from $\frac{dr}{d\theta} = \frac{1 - C_{r\theta}}{C_{rr}}$ and assumption 1. $\partial P^i / \partial \theta > 0$ is fulfilled if the derivative of (5) with respect to θ is positive. Focusing on the critical case where $r^s > 0$, we need $\tau^{nc} - \tau^c + C_\theta(r^{nc}, \theta) - r^{nc} + \tau^c r^s - C_\theta(r^s, \theta) > 0$. Using $C_\theta(r^{nc}, \theta) = C_0 + \int_{r=0}^{r=r^{nc}} C_{\theta r} dr$ and $C_\theta(r^s, \theta) = C_0 + \int_{r=0}^{r=r^s} C_{\theta r} dr$ we get $C_\theta(r^{nc}, \theta) - C_\theta(r^s, \theta) = \int_{r=r^s}^{r=r^{nc}} C_{\theta r} dr = r^{nc} - r^s + L$ with some $L \geq 0$. So we can rewrite the condition $\tau^{nc} - \tau^c - r^s + \tau^c r^s + L > 0$ which is fulfilled for $r^s < \frac{\tau^{nc} - \tau^c}{(1 - \tau^c)}$, i.e. assumption 3 (a).

C. Critical mass of evaders: Proof of lemma 3

Let $\phi := \theta | (\delta(\theta, k_t) = 0)$. Implicitly differentiating $\delta(\phi) = 0$ gives $\frac{d\phi}{dk_t} = (1 - \beta \frac{\partial (V_T^c - V_T^{nc})}{\partial k_t}) / (\frac{d\delta}{d\phi}) > 0$ where $\beta \frac{\partial (V_T^c - V_T^{nc})}{\partial k_t} < 1$ and $\frac{d\delta}{d\phi} > 0$ by lemma 2. That $\phi(\overline{k}) = 1$ and $\phi(\underline{k}) = \underline{\theta}$ follows from lemma 1.

D. Proof of proposition 2

1. Thresholds

Signal x^i of k_t which citizen i receives is equally distributed over $(k_t - \varepsilon, k_t + \varepsilon)$. A citizen i' strategy has the form: evade taxes if $x^i \leq \xi^i$ for some cut off point ξ^i. For the moment, assume that the distribution of cut off points is exogenously given according to $\widetilde{\xi}$ with $f(\xi) : \xi(i) \rightarrow \Re^+$. If k_t is the true state, then the probability that $x \leq \xi$ is given by

$$W(\widetilde{\xi}|k_t) = \int_{x=k_t-\varepsilon}^{x=k_t+\varepsilon} \frac{1}{2\varepsilon} \int_{\xi=x}^{\xi=\infty} f(\xi) d\xi dx. \tag{11}$$

$W(\widetilde{\xi}|k_t)$ is the share of citizens who have received a signal falling below their individual cut off point ξ given that $\widetilde{\xi}$ is distributed according to f. The term on the right hand side gives the probability that ξ is higher than the signal in the interval $[k_t - \varepsilon, k_t + \varepsilon]$. Now, if the true state is k_t, then the government switches back with probability one if $W(\widetilde{\xi}|k_t) > \phi(k_t)$. The minimum k_t for which the government does not switch back is uniquely given by

$$k_t' = \min\{k_t | W(\widetilde{\xi}|k_t) \leq \phi(k_t)\}. \tag{12}$$

Now, the probability which an agent who receives the message x^i assigns to the event that the government switches back is

$$\psi^i(W(k_t, \widetilde{\xi}^{-i}), \phi(\theta)|x^i) = \int_{x^i-\varepsilon}^{\min(k_t', x^i+\varepsilon)} \frac{1}{2\varepsilon} dk_t \tag{13}$$

where $\frac{1}{2\varepsilon}$ is the density of the distribution of k_t and $\widetilde{\xi}^{-i}$ is the distribution of ξ without the agent i (which coincides with $\widetilde{\xi}$ because the agent is atomic). We get ψ by integrating over all k_t which are in accordance with a defection by the government and relating them to all k_t which are possible from the observation (which has measure 1). Let ξ^i be the highest signal x^i which elicits the reaction of a citizen, i.e. for which $\psi^i(W(k_t,\widetilde{\xi}^{-i}),\phi(k_t)|x^i)$ satisfies (1) as an equality or where P^i assumes its critical value P^{i*}:

$$\xi^i = Max\{x^i| \int_{x^i-\varepsilon}^{\min(k'_t,x^i+\varepsilon)} \frac{1}{2\varepsilon} dk_t \geq P^{i*}\}. \tag{14}$$

We obtain k^* and $\widetilde{\xi}(k^*)$ as the limit of iteratively eliminating weakly dominated strategies starting at the interval borders with $\widetilde{\xi}_0^b = \underline{k} - \varepsilon$ and $\widetilde{\xi}_0^u = \overline{k} + \varepsilon$.

2. Uniqueness of k^*

First, we establish that for any switching point k^* there is a unique distribution $\widetilde{\xi}$ such that (14) holds for every agent. Note that $f(\xi)$ is common knowledge. Let $F(\xi)$ be the cumulative distribution of $f(\xi)$ so that (11) can be represented as $W(\widetilde{\xi}|k) = \int_{x=k_t-\varepsilon}^{x=k_t+\varepsilon} \frac{1}{2\varepsilon}(1-F(x))dx$. Let ξ fulfill (14). Suppose there is $\widetilde{\xi}'$ and agent i^L such that $\xi'(i^L) > \xi(i^L)$. In order to fulfill $W(\widetilde{\xi}'|k^*) = \phi(k^*)$ at the new distribution, $F(x)$ and $F'(x)$ must cross at least once for some $\xi(i) < \xi^{\overline{\eta}}$. Say i^L is on the left hand side of the first such crossing so at the crossing, $F(\xi')$ cuts $F(\xi)$ from below. Let i^C be the agent located at the crossing (i.e. for whom $\xi'(i^C) = \xi(i^C)$ and i^R an agent on the right hand side of the first and to the left of a second crossing (if it exists) with $\xi'(i^R) < \xi(i^R)$. Assume that (14) holds for i^C. From (14), $\psi(\xi'(i^L))/\psi(\xi'(i^R)) < \psi(\xi(i^L))/\psi(\xi(i^R))$. Because $\widetilde{\xi}$ satisfies (14) for i^L and i^R, $\widetilde{\xi}'$ does not.

To proof uniqueness of k^*, suppose there is another cut off point $k' < k^*$ with $\phi(k') < \phi(k^*)$. We construct the new (and unique) system of threshold values $\widetilde{\xi}'$ in two steps: First, calculate $\widetilde{\xi}''$ as an exact translation of $\widetilde{\xi}$ by letting $\xi'' = \xi - k^* + k'$.

Calculate the subjective probabilities with ξ'' assuming that the critical value is as before $\phi(k^*)$, i.e. $\psi^i(W(k',\widetilde{\xi}''),\phi(k^*)|\xi^i) = \int_{\xi^i-\varepsilon}^{k'} \frac{1}{2\varepsilon} dk$. By construction, this system of probabilities satisfies again (14) for each i. Because $\phi(k') < \phi(k^*)$ we know that

$$\psi^i(W(k', \widetilde{\xi}''), \phi(k')|x^i) > \psi^i(W(k', \widetilde{\xi}''), \phi(k^*)|x^i),$$

for all x^i. In order to fulfill (14) with the true values ξ' and $\phi(k')$, ψ^i needs to be lowered, i.e.

$$\psi^i(W(k', \widetilde{\xi}''), \phi(k')|\xi^{i\prime\prime}) > \psi^i(W(\theta', \widetilde{\xi}'), \phi(k')|\xi^{i\prime})$$

Because ψ^i decreases in ξ^i it must be that $\xi^{i\prime} > \xi^{i\prime\prime}$ for all i. Now suppose that k' is the true value. Then the set of evaders $\theta(k')$ has increased compared to the system $\widetilde{\xi}''$. With $\widetilde{\xi}''$ we have $\theta(k') = W(\widetilde{\xi}''|k') = \phi(k^*) > \phi(k')$ because $\widetilde{\xi}''$ is an exact translation of $\widetilde{\xi}$. With $\widetilde{\xi}'$, we have $W(\widetilde{\xi}'|k') > W(\widetilde{\xi}''|k')$ because all individual cut off point have moved to the right and more agents evade for any given signal. Thus $\theta(k') > \phi(k')$ contradicting that k' is a switching point.

E. Comparative statics

For $\varepsilon \to 0$, $\underline{\eta} \to \eta^0$, $\overline{\eta} \to \eta^0$ we can apply the results of Heinemann (2000). In equilibrium it must hold that $P(1 - \widehat{r}^{si})\tau^c + (1 - P)(\tau^{nc} - \widehat{r}^{nci}) = \eta^0$. Using $\widehat{r}^{si} = r^s$, $\widehat{r}^{nci} = r^{nc}$ and $P = 1 - \phi(k^*)$ for $\varepsilon \to 0$ gives the equilibrium condition

$$\frac{1 - \phi(k^*)}{\phi(k^*)} = \frac{\frac{\eta^0}{\phi(k^*)} - (\tau^{nc} - r^{nc})}{(1 - r^s)\tau^c}.$$

The left hand side of this equation goes from ∞ to 0 as k^* goes from $\underline{k}$ to $\overline{k}$ and has a graph which is convex to the origin. The degree of convexity of the graph directly relates to the convexity of the cost function. Therefore, k^* is closer to $\underline{k}$ the greater the value of the right hand side of the equation.

F. Proof of proposition 3

Before we proof the proposition, the following lemma is useful:

Lemma 5 *(Approximate observations) Citizens correctly forecast the share of tax evaders if the noise in the observation vanishes* ($\varepsilon \longrightarrow 0$).

Proof. From (11) we know that the share of agents who receive a signal short of their threshold or - equivalently - the amount of evaders is $W(\widetilde{\xi}|k) = \int_{x=k-\varepsilon}^{x=k+\varepsilon} \frac{1}{2\varepsilon} \int_{\xi=x}^{\xi=\infty} f(\xi) d\xi dx$ if the true value is k. Now $\Omega(\widetilde{\xi}|x) = \int_{x-\varepsilon}^{x+\varepsilon} \frac{1}{2\varepsilon} W(\widetilde{\xi}|k) dk$ is the expected share of tax evaders if the observation is x. Taking the limit for vanishing ε gives $\underset{\varepsilon \to 0}{lim}\ \Omega(\widetilde{\xi}|x) = W(\widetilde{\xi}|k)$. ■

$\widehat{r}^{nci}$ is defined as citizen i's expected detection rate conditional on the government staying outside the constitution while $\widehat{r}^{si}$ is the expected detection rate conditional on the government switching back to the constitution. If a citizen receives the signal x^i of k_t and the government stays outside the constitution if $k_t > k^*$ then this expectation is

$$\widehat{r}^{nci}(x^i) = \frac{1}{x^i + \varepsilon - k^*} \int_{k^*}^{x^i+\varepsilon} r^{nc}(k_t) dk_t$$

$$\widehat{r}^{si}(x^i) = \frac{1}{k^* - x^i + \varepsilon} \int_{x^i-\varepsilon}^{k^*} r^{s}(k_t) dk_t$$

Using $\sigma = 1$ in (1), citizen i evades taxes if for some P^i

$$(1 - P^i)(\tau^{nc} - \widehat{r}^{nci}(x^i)) + P^i(1 - \widehat{r}^{si}(x^i))\tau^c \geq \eta^i.$$

Using $\tau^c(1 - \widehat{r}^{si}) > \overline{\eta}$ and $\overline{\eta} > \eta^i$ from assumptions 1 and 4 (b), i evades if

$$(1 - P^i)(\tau^{nc} - \widehat{r}^{nci}(x^i)) \geq (1 - P^i)(1 - \widehat{r}^{si}(x^i))\tau^c. \tag{15}$$

Now consider the position of the most willing tax payer, the citizen with $\eta^i = \overline{\eta}$. The threshold for this citizen must be $\underline{\xi} = k^* - \varepsilon$: If she gets a signal $x^i \leq \underline{\xi}$ she sets $P^i = 1$ and evades by assumptions 1 and 3. Next, assume that the same citizen receives some infinitesimally greater signal, $x^i = \underline{\xi}^+$ so that at this signal (15) is fulfilled. Now, $P^i < 1$ but the citizen still evades. Consider higher signals $x^i > \underline{\xi}^+$. As $\frac{dr}{d\theta} < 0$ and higher values of k_t come with lower tax evasion, at these signals, $\widehat{r}^{nci}(x^i) > \widehat{r}^{nci}(\underline{\xi}^+)$ and $\widehat{r}^{si}(x^i) > \widehat{r}^{si}(\underline{\xi}^+)$. So the only reason why $\overline{\eta}$ would ever pay taxes could be that with higher signals the conditional expectation $\widehat{r}^{nci}$ increases sufficiently more than $\widehat{r}^{si}$. Next consider a citizen with $\xi' > \underline{\xi}$. For this citizen $\widehat{r}^{nci}(\xi') > \widehat{r}^{nci}(\underline{\xi})$ and $\widehat{r}^{si}(\xi') > \widehat{r}^{si}(\underline{\xi})$. So if (15) is fulfilled for the citizen with $\underline{\xi}$, it is also fulfilled for the citizen with ξ' unless her expectation of $\widehat{r}^{nci}$ were to increase sufficiently more than $\widehat{r}^{si}$. Now let $\varepsilon \to 0$. Lemma 5 establishes that $\widehat{r}^{nci}(\xi') \to \widehat{r}^{nci}(\underline{\xi})$ and $\widehat{r}^{si}(\xi') \to \widehat{r}^{si}(\underline{\xi})$. But then if $\tau^{nc} - \widehat{r}^{nci}(x^i) > (1 - \widehat{r}^{si}(x^i))\tau^c$ all citizens, irrespective of their belief P^i would evade taxes.

G. Proof of lemma 4

For $x^i \in (k_{t-1} - \Delta + \varepsilon, k_{t-1} + \Delta - \varepsilon)$ the prior k_{t-1} does not affect the posterior distribution. So $\widehat{k}^* \neq k^*$ only for $x^i \in (k_{t-1} - \Delta, k_{t-1} - \Delta + \varepsilon)$ and $x^i \in (k_{t-1} + \Delta - \varepsilon, k_{t-1} + \Delta)$ but both intervals vanish as $\varepsilon \to 0$.

H. Proof of proposition 4

Note that in both periods, $k^0 = z^c - z^{nc}$ and, in particular, $k^{**\prime} = k^0$. Applying (5) we get $-z^{nc} - k^* + \beta(z^c - z^{nc} - k^*) = 0$ or $k^* = \frac{\beta}{1+\beta} z^c - z^{nc}$. Because $k^0 = z^c - z^{nc}$, this implies $k^* < k^0$. At $k^{**} < k^* + \Delta$ we get $z^c = \int_{k_t \geq k^*} \omega(k_t|k^{**})(z_t^{nc} + k_t)dk_t - \beta \int \omega D_{t+1} dk_t < z_t^{nc} + k^{**}$

contradicting $k^{**} \geq k^0$. Therefore, $k^{**} < k^* + \Delta$ implies $k^{**} < k^0$.[18] For $k^{**} \geq k^* + \Delta$, $k^{**} = k^0$. It remains to give a condition under which $k^0 \leq k^* + \Delta$. Using the closed-form expressions for k^* and k^0 we immediately get $\frac{1}{1+\beta} z^c \leq \Delta \Longleftrightarrow k^0 \leq k^* + \Delta$.

I. Proof of proposition 5

In this proof we proceed as follows: Lemma 6 and 7 extend the uniqueness result on k^* of lemma 1 to the infinite horizon model. Subsequently we show that there uniquely exists a fixed point $k^{**} = k^{**\prime}$ for which $D(k) = 0$. This proves the proposition.

Lemma 6 *There is $\widehat{k}$ such that $D(k)$ decreases for all $k > \widehat{k}$.*

Proof: The derivative of (10) with respect to k_{t-1} for $k^{**\prime} > k^*$ and $k^{**\prime} < k^* + 2\Delta$ is [19]

$$
\begin{array}{ll}
k' < k^* - \Delta : & \frac{\partial D_t(k')}{\partial k_{t-1}} = 0. \\
k' \in [k^* - \Delta, k^{**\prime} - \Delta) : & \frac{\partial D_t(k')}{\partial k_{t-1}} = -\omega\left(z^{nc} + k' + \Delta\right) + \beta\omega D_{t+1}(k' + \Delta) \leq 0. \\
k' \in [k^{**\prime} - \Delta, k^* + \Delta) : & \frac{\partial D_t(k')}{\partial k_{t-1}} = -\omega\left(z^{nc} + k' + \Delta\right) < 0. \\
k' \in [k^* + \Delta, k^{**\prime} + \Delta) : & \frac{\partial D_t(k')}{\partial k_{t-1}} = -1 - \beta\omega D_{t+1}(k' - \Delta) < 0 \\
k' \geq k^{**\prime} + \Delta : & \frac{\partial D_t(k')}{\partial k_{t-1}} = -1
\end{array}
$$

For $k^{**\prime} < k^*$ we rewrite (10) to give

$$
D_t(k_{t-1}) = z^c - \int_{\max(k^*, k_{t-1}-\Delta)}^{k_{t-1}+\Delta} \omega(z^{nc} + k_t)dk_t - \beta \int_{\max(k^{**\prime}, k_{t-1}-\Delta)}^{\min(k^*, k_{t-1}+\Delta)} \omega D_{t+1}(k_{t-1})dk_t \qquad \text{(10a)}
$$

with the following derivatives:[20]

[18] There is a unique solution for k^{**} because $D_{T-1}(k_{T-2}) > 0$ for $k_{T-2} \leq k^* - \Delta$ and decreasing in k_{T-2} for $k_{T-2} > k^* - \Delta$ (see lemma 6 in the proof of proposition 5).

[19] In the case $k^{**\prime} \geq k^* + 2\Delta$ the second derivative holds in $k' \in [k^* - \Delta, k^* + \Delta)$, for $k' \in [k^* + \Delta, k^{**\prime} - \Delta)$ we have $\frac{\partial D_t(k')}{\partial k_{t-1}} = - - 1 + \beta\omega D_{t+1}(k' + \Delta) < 0$, for $k' \in [k^{**\prime} - \Delta, k^{**\prime} + \Delta)$ we have $\frac{\partial D_t(k')}{\partial k_{t-1}} = -1 - \beta\omega D_{t+1}(k' - \Delta) < 0$ and the fifth derivative holds for $k' \geq k^{**\prime} + \Delta$.

[20] In the case $k^{**\prime} < k^*$ the relationship $k^* - k^{**\prime} < \Delta$ must hold: Otherwise the probability of succeeding in a defection would be zero at $k^{**\prime}$.

$k' < k^{**\prime} - \Delta :$ $\quad \frac{\partial D_t(k')}{\partial k_{t-1}} = 0.$

$k' \in [k^{**\prime} - \Delta, k^* - \Delta) :$ $\quad \frac{\partial D_t(k')}{\partial k_{t-1}} = -\beta\omega D_{t+1}(k' + \Delta) > 0.$

$k' \in [k^* - \Delta, k^{**\prime} + \Delta) :$ $\quad \frac{\partial D_t(k')}{\partial k_{t-1}} = -\omega\left(z^{nc} + k' + \Delta\right) \lesseqgtr 0.$

$k' \in [k^{**\prime} + \Delta, k^* + \Delta) :$ $\quad \frac{\partial D_t(k')}{\partial k_{t-1}} = -1 + \beta\omega D_{t+1}(k' - \Delta) < 0$

$k' \geq k^* + \Delta$ $\quad \frac{\partial D_t(k')}{\partial k_{t-1}} = -1$

Noting that $D_{t+1}(k_t) > 0$ for $k_t < k^{**\prime}$ and switches signs in $k^{**\prime}$, the claim follows immediately in the case $k^* < k^{**\prime}$ with $\widehat{k} = k^* - \Delta$. For $k^* > k^{**\prime}$, there is $\widehat{k} \in [k^* - \Delta, k^{**\prime} + \Delta)$ for which $\frac{\partial D_t(k')}{\partial k_{t-1}}$ becomes negative.■

Lemma 7 *In the stationary game and $k^* > \widehat{k}$.*

Proof: We have to show that at $k' = k^*$, $\frac{\partial D_t(k')}{\partial k_{t-1}} < 0$. Applying (10a) to period $t+1$ we get $D_{t+1}(k^{**\prime}) = z^c - \int_{k^{*\prime}}^{k^{**\prime}+\Delta} \omega(z^{nc} + k_{t+1})dk_{t+1} - \beta \int_{k^{**\prime\prime}}^{k^{*\prime}} \omega D_{t+2} dk_{t+1}$. At k^*, we have $D_{t+1}(k^*) - z^c + \int_{k^{*\prime}}^{k^*+\Delta} \omega(z^{nc} + k_{t+1})dk_{t+1} = -\beta \int_{k^{**\prime\prime}}^{k^{*\prime}} \omega D_{t+2} dk_{t+1}$. Substituting the last term, we get

$$D_{t+1}(k^*) = D_{t+1}(k^{**\prime}) - \int_{k^{**\prime}+\Delta}^{k^*+\Delta} \omega(z^{nc} + k_{t+1})dk_{t+1}.$$

By definition $D_{t+1}(k^{**\prime}) = 0$. $|D_{t+1}(k^*)|$ increases in the distance $k^* - k^{**\prime}$. Because we know (see footnote 17) that this distance is smaller Δ we set $k^* - k^{**\prime} = \Delta$ and solve the integral using $\omega = \frac{1}{2\Delta}$. This gives $D_{t+1}(k^*) = -\frac{1}{2}z^n - \frac{1}{2}k^* - \frac{1}{4}\Delta$. Inserting into (5) gives $-z^n - k^* - \beta(\frac{1}{2}z^n + \frac{1}{2}k^* + \frac{1}{4}\Delta) = 0$ from which we conclude $-z^n - k^* - \frac{\beta}{2+\beta}\Delta = 0$. Therefore, $z^{nc} - k^* - \Delta < 0$. Inserting into the expression for $\frac{\partial D_{t+1}(k')}{\partial k_t}$ shows that the derivative is negative at $k_t = k^*$. In the stationary game, $D_t(k') = D_{t+1}(k')$ and D_t is decreasing at $k_{t-1} = k^*$.■

Lemma 8 *In a stationary game the switching point $k^* > \widehat{k}$ is unique.*

Proof: By lemma 7, $k^* > \widehat{k}$ and D_{t+1} decreases in $(\widehat{k}, \infty)$. Now let $\Delta > 2\varepsilon \geq |\overline{k} - \underline{k}|$. We have to show that $\overline{k} > \underline{k}$. Suppose that $\overline{k} \leq \underline{k}$. Because D_{t+1} is decreasing in k, $D_{t+1}(\overline{k}) \geq D_{t+1}(\underline{k})$ follows. But in that case, by lemma 1 it must be $\overline{k} > \underline{k}$, a contradiction. Therefore, $\overline{k} > \underline{k}$ and $\widehat{k}$ is a switching point by proposition 2 and this point is unique in $(\widehat{k}, \infty)$.■

Let $X = k^{**\prime}$ and $Y = k^{**}$. Using (10) and (10a) we can now implicitly define the mapping Γ: $R \rightarrow R$ as follows:

For $X < k^*$:

$$h(X,Y) \equiv z^c - \beta \int_{\max(X,Y-\Delta)}^{\min(k^*,Y+\Delta)} \omega D_{t+1} dk - \int_{k^*}^{Y+\Delta} \omega(z^{nc} + k_t) dk_t = 0. \tag{16}$$

and for $X \geq k^*$:

$$h(X,Y) \equiv z^c + \beta \int_{\max(k^*,Y-\Delta)}^{\min(X,Y+\Delta)} \omega D_{t+1} dk - \int_{\max(k^*,Y-\Delta)}^{Y+\Delta} \omega(z^{nc} + k_t) dk_t = 0. \tag{17}$$

It is straightforward that for all $X \leq k^* - \Delta$ it must be that $Y > k^* - \Delta$: Suppose that $Y \leq k^* - \Delta$. Then we get $D_t(Y) = z^c - \int_{k^*}^{\min(k^*,Y+\Delta)} \omega(z^{nc} + k) dk - \beta \int_X^{\min(k^*,Y+\Delta)} \omega D_{t+1} dk > 0$. The inequality sign follows because the first integral is zero and is followed by a term which is positive with $D_t(k) < 0$ for $k > X$. $D_t(Y) > 0$ contradicts that Y is a defection point. Moreover, D_{t+1} is finite for $X \rightarrow \infty$ for all Y because z^c is finite and $\beta < 1$. Therefore, $\Gamma(X) < \infty$. Γ has a fixed point if the mapping is continuous. For $X \rightarrow k^*$ we have $D_t(k')|k^* < X \rightarrow D_t(k')|X = k^*$, so Γ is continuous.

Finally, we have to show that the fixed point is unique. Differentiating expression[21] (16) above at $Y = X$ we get $\frac{dY}{dX} = \frac{-h_X}{h_Y} = \frac{\beta\omega D_{t+1}(X)}{-\omega(z^n+Y+\Delta)}$ (in the case (17) $\frac{dY}{dX} = \frac{-\beta\omega D_{t+1}(X)}{-\omega(z^n+Y+\Delta)}$). Using $D_{t+1}(X) = 0$, we see that the graph of Γ crosses the $Y = X$ line from above. Because Γ is also continuous, the fixed point is unique.

VIII. Appendix 2: The dynamic tax problem

In this appendix we describe the tax-rate-selection problem in the beginning of period t in more detail. The government has to determine outcomes depending on the realization of k_t when it knows k_{t-1}. Dropping the discount factor and defining conditional expectations $E_{|k_t \leq k^*}$ for conditional on $k_t \leq k^*$ with probability $\pi_{|k_{t-1}} = \int_{k^*}^{k_{t-1}+\Delta} \omega(k_t)dk_t$ and $E_{|k_t > k^*}$ for $k_t > k^*$ with probability $(1 - \pi_{|k_{t-1}})$ and using $\theta(.) = \theta(\tau^{nc}, \tau^c, \widetilde{r}^{nc}, \widetilde{r}^s, \widetilde{P})$ we get

$$\begin{aligned} \underset{\tau^{nc}}{Max}\ \ E_{|k_{t-1}}z &= (1 - \pi_{|k_{t-1}})E_{|k_t > k^*}[\theta(.)r^{nc}(\theta) + (1 - \theta(.))\tau^{nc} - C(r^{nc}(\theta), \theta(.)) + \beta V_{t+1}^{nc}] \\ &\quad + \pi_{|k_{t-1}}E_{|k_t \leq k^*}[\theta(.)\tau^c r^s(\theta) + (1 - \theta(.))\tau^c - C(r^s(\theta), \theta(.)) + \beta V_{t+1}^c] \text{ s.t. } \tau^{nc} \leq 1 \end{aligned}$$

We solve the dynamic tax problem for the case where the covariance between the marginal propensity to evade taxes and the marginal cost of detection disappears and the noise in the citizens' observations vanishes. For $\varepsilon \to 0$ the government simply maximizes revenue in the non constitutional state so the optimal tax rate τ^{nc*} coincides with the ex post optimal tax rate τ^0. This is because the selection of the tax rate has no impact on citizens' behavior for $k_t \leq k^*$ so τ^0 also minimizes the probability of a switch back.

Define the Lagrangian $\pounds = E_{|k_{t-1}}z + \beta V + \lambda(1 - \tau^{nc} - s)$ for some $s \geq 0$ where λ is

[21] k^* does not depend on X: X enters (5) through the term $D_{t+1}(k_t, X) = z^c - \int_{k^{*\prime}} \omega(z^{nc} + k_{t+1})dk_{t+1} + \beta \int_{k^{*\prime}}^{X} D_{t+2}dk_{t+1}$. As $D_{t+2}(X) = 0$ we have $\frac{\partial D_{t+1}}{\partial X} = 0$ from which the claim follows.

the shadow price of the constraint $\tau^{nc} \leq 1$. From the tax-detection equilibrium (2) and (4) we obtain policy functions for the case $k_t \leq k^*$ with $\frac{d\theta}{d\tau}_{|nc} = \frac{-C_{rr}}{\det}$ and $\frac{dr}{d\tau}_{|nc} = \frac{C_{r\theta}-1}{\det}$ with $\det = \frac{-C_{rr}}{\theta_\tau} + \frac{\theta_r}{\theta_\tau}(1 - C_{r\theta}) < 0$ by assumption 1. Therefore, $\frac{d\theta}{d\tau}_{|nc} > 0$ and $\frac{dr}{d\tau}_{|nc} < 0$. In a similar way we can derive marginal responses $\frac{d\theta}{d\tau}_{|s}$ and $\frac{dr}{d\tau}_{|s}$ conditional on $k_t \leq k^*$

Assuming the covariance between θ_τ and marginal cost disappears, the first order and Kuhn-Tucker conditions for this problem are

$$\frac{\partial \pounds}{\partial \lambda} = 1 - \tau^{nc} - s \geq 0,\ s \geq 0,\ \lambda \geq 0,\ s\lambda = 0, \tag{18}$$

$$\begin{aligned} \frac{\partial \pounds}{\partial \tau} &= (1-\pi)E_{|k_t>k^*}[(1-\theta) - (\tau^{nc} - r^{nc} + C_\theta)\frac{d\theta}{d\tau}_{|nc} + (\theta - C_r)\frac{dr}{d\tau}_{|nc}] \\ &+ \pi E_{|k_t \leq k^*}\left[(-\tau^c - C_\theta)\frac{d\theta}{d\tau}_{|s} + (\tau^c\theta - C_r)\frac{dr}{d\tau}_{|s}\right] \\ &+ \frac{\partial \pi}{\partial \tau}\left[-E_{|k_t>k^*}z^{nc} - k_t - D_{t+1} + E_{|k_t \leq k^*}z^s\right] - \lambda \geq 0 \end{aligned} \tag{19}$$

with $D_{t+1} = \beta(V^c_{t+1} - V^{nc}_{t+1})$ and the conditional policy derivatives obtained above.

The first line of the right hand side of the equation (19) is the expected effect on non-constitutional receipts, the second is the expected effect on the switch back income. For $k_t > k^*$ we either have $\tau^c\theta(k_t) - C_r(r^{nc}, \theta(k_t)) = 0$ or $dr = 0$ so the term involving $\frac{dr}{d\tau}_{|nc}$ disappears and $\frac{d\theta}{d\tau}_{|nc} > 0$. The second line is non positive and the term involving $\frac{dr}{d\tau}_{|s}$ disappears. By lemma 5, for $\varepsilon \to 0$, all citizens set $P^i = 1$ and $\widehat{r}^{si} = r^s$ at $k_t \leq k^*$ and we have $z^s = 0$. In that case, the partial derivative of the tax evasion function with respect to the level of taxes is $\frac{\partial\theta(.)}{\partial\tau} = 0$. Therefore, the marginal responses conditional on $k \leq k^*$, $\frac{d\theta}{d\tau}_{|s}$ and $\frac{dr}{d\tau}_{|s}$ vanish and the second line is zero. In the third line we have the effect of the tax rate on the switch back probability $\frac{\partial\pi}{\partial\tau} = \frac{1}{2\Delta}\frac{dk^*}{d\tau}$. We evaluate $\frac{dk^*}{d\tau}$ at $k^{*0} + \partial k^*$, so we get $\frac{dk^*}{d\tau} = \frac{dz^s}{d\tau} - \frac{dz^{nc}}{d\tau}$ with $\frac{dz^s}{d\tau} = (\tau^c(r^s - 1) - C_\theta)\frac{d\theta}{d\tau}$. So $\frac{dz^s}{d\tau} \leq 0$ for $\frac{d\theta}{d\tau} \geq 0$. Again using $\varepsilon \to 0$

with $z^s(k^*) = 0$ and $sgn\frac{\partial\pi}{\partial\tau} = sgn\frac{dk^*}{d\tau} = -sgn\frac{dz^{nc}}{d\tau}$ so we have $\frac{\partial\pi}{\partial\tau} = 0$ at the point where revenue in the non constitutional state, z^{nc}, is maximized. Therefore, $\tau^{nc*} = \tau^0$. The term in brackets in the third line is the cost of not committing with negative sign: for a defecting government, it must be true that $(1-\pi)E_{|k>k^*}[(z^{nc} + k_t + D_{t+1}] + \pi E_{|k\leq k^*} z^s \geq z^c$. But as $E_{|k\leq k^*} z^s < z^c$, it follows that $E_{|k>k^*}(z^{nc} + k_t + D_{t+1}] > E_{|k\leq k^*} z^s$.